## Special Feature: Active Safety

## Research Report

# A Computational Framework for Estimating Collision Risk against Pedestrians 

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#### Abstract

IABSTRACTII In the present paper, a new framework for calculating subjective risk estimation (SRE) of an attentive human driver is proposed. First, we conduct a psychological interview of an expert driver to obtain his cognitive structure of SRE. These results suggest that SRE can be expressed as a collision probability between the ego-vehicle and other road users, and the probability can be estimated based on subjective behavior prediction (SBP) for the road users, which is the future position of each user expressed as a probability distribution (position distribution). The future position distribution is influenced by two types of subjective factors on a traffic scene: environmental factors and target factors. These factors can prompt and regulate SBP for each road user. Second, the cognitive structure of SRE is mathematically redescribed to propose the framework for SRE. Third, we propose an algorithm by which to calculate SRE, which is a collision probability, using a particle filter technique. Finally, numerical simulations for several traffic situations, in which the ego-vehicle passes a pedestrian, are conducted. The results of these simulations reveal the validity of the proposed framework.


## KEYWORDS|| Pedestrian Behavior, Collision Probability, Risk Estimation, Probabilistic Model, Particle Filter, Risk Estimation

## 1. Introduction

Recently, preventive safety systems such as Forward Collision Warning (FCW) ${ }^{(1)}$ and Autonomous Emergency Braking (AEB) ${ }^{(2)}$ have been made commercially available. Their effectiveness in accident reduction has been verified by several studies. ${ }^{(3,4)}$ However, current systems cannot always avoid collisions because they are designed to perform their functions only when a collision with a target object, e.g., a leading vehicle or a pedestrian, in the planned path is unavoidable.
In order to improve current systems, one promising method is to take into account the safe driving behaviors of an attentive human driver. The driver can avoid hazardous situations by reducing speed or preparing to brake in case of a sudden change in the traffic situation, whether or not this change actually occurs. In order to drive in such a safe manner, it is essential to predict changes in future traffic and to estimate the collision risks. Even if potentially hazardous road users are not in the driver's planned path, the driver predicts their future behaviors and
interactions and estimates the risk of collision with each of these users so that the driver can control the vehicle speed appropriately and maintain safe distances from these users. Technically realizing behavior prediction and risk estimation of the driver could lead to improved current systems.
It is important to note that behavior prediction and risk estimation of the driver are subjective. Moreover, subjective behavior prediction (SBP) and subjective risk estimation (SRE) could be affected by various factors related to traffic. For example, the presence of a road marking affects the speed at which drivers approach a pedestrian at a crosswalk. ${ }^{(5)}$ Another study ${ }^{(6)}$ has shown that drivers identify a pedestrian's intention to cross a road from the pedestrian's posture. These results imply that the driver could use some features of the road environment and the pedestrian's state to predict the pedestrian's behavior and to estimate the risk. In the present paper, the focus is on proposing a new computational framework for SBP and SRE.
In recent years, behavior prediction frameworks for road users have been investigated in terms of road
safety using various approaches. Pattern learning approaches ${ }^{(7,8)}$ apply machine learning techniques to predict future trajectories of traffic users at a fixed location. These approaches are based on real world traffic data and are quite objective. The knowledgebased approach ${ }^{(9,10)}$ attempts to structuralize hazardous situations. Although this approach could potentially use any types of features in a traffic scene to assess risk, ${ }^{(11)}$ the overall capability of this approach is limited to describing the temporal and spatial relationships between multiple road users. In contrast, the Bayesian approach ${ }^{(12-14)}$ has the advantage of capturing complex interactions between multiple road users in future traffic situations. However, in previous studies based on this approach, only physical constraints are considered to predict the possible future behavior of each road user. The physical constraints are not sufficient parameters for SBP.
In the present paper, a framework for calculating SRE is proposed. SRE is expressed as a collision probability obtained on the basis of SBP, which is the future position of each road user expressed as a probability distribution (position distribution). The position distribution is obtained using the particle filter technique. ${ }^{(15)}$ In order to incorporate the subjective effects in the particle filtering, two specific parameters, both of which are determined in a subjective manner, are introduced as random noise in the update step and the resampling weight in the resampling step. The remainder of the present paper is organized as follows. Section 2 structuralizes SRE through a psychological interview of an expert driver to clarify the cognitive relationship between SRE and SBP. The relationship is then described mathematically in order to propose the framework for SRE. Section 3 describes the algorithms used to calculate SBP and SRE for numerical experiments conducted in Section 4. Finally, we present the conclusions in Section 5.

## 2. Framework for SRE

## 2. 1 Schematic Framework for SRE

We conducted a psychological interview of a male expert driver to obtain a comprehensive structure by which to derive a framework for SRE. This driver is regarded as an attentive driver because he works as a safe driving instructor in our laboratory. The evaluation grid method, ${ }^{(16)}$ which is a modified repertory grid technique, ${ }^{(17)}$ was used as the interview method. In the
interview, using printed pictures of traffic scenes selected from near-miss incident data, an interviewer repeatedly asked the driver why traffic scenes are hazardous and what are the hazards in the scenes.

The results of the interview are the structure of the driver's risk perception. Figure 1 illustrates one part of the structure. (The entire structure is too large to fit on one page.) In the figure, the sentences inside the solid boxes are the driver's answers, and the texts outside the boxes are our interpretations. These results provide insight into the framework of SRE, as follows:
a) The expert driver subjectively estimates the possibility of collision between the ego-vehicle and another road user as SRE for a traffic scene.
b) The possibility of collision is estimated, i.e., SRE is performed, based on SBP for both the ego-vehicle and other road users.
c) SBP has an uncertainty caused by two types of factors, namely, target factors and environmental factors:
Target factors are subjective factors concerning other road users, which result in uncertainty in the behavior prediction of the driver. These factors include posture, age, and motions indicating inattentiveness.

Environment factors are subjective factors concerning other unspecified road users that make up the traffic environment, which result in uncertainty in the behavior prediction of the driver. These factors include the presence of crosswalks, the absence of sidewalks, weather, and the number of other road users.


Fig. 1 Part of the structure of the subjective risk estimation.

Figure 2 is a schematic diagram for the framework of SRE. SRE can be expressed by a collision probability between the ego-vehicle and other road users, which is derived based on SBP for each road users. SBP should be determined in a stochastic manner rather than in a deterministic manner because SBP must have an uncertainty caused by target factors and environmental factors. In the present paper, SBP is expressed as a probability distribution of the position of each road user.

## 2. 2 Mathematical Framework for SRE

In this section, a mathematical framework for SRE is described as a Bayesian representation. Let $C_{T}$ denote a binary probability variable describing the collision state of the ego-vehicle (driver's vehicle) at time $T$. If no collision occurs until time $T, C_{T}=0$; otherwise, $C_{T}=1$. Taking the output of the outsidemonitoring sensor at time 0 (the current time) as $S_{0}=$ $s_{0}$, the collision probability that the ego-vehicle collides with at least one other road user by time $T=t_{f}$ can be defined as follows:

$$
\begin{align*}
p\left(C_{T}=1 \mid C_{0}=0,\right. & \left.S_{0}=s_{0} ; T=t_{f}\right) \\
& =1-\prod_{T=1}^{t_{f}} p\left(C_{T}=0 \mid C_{T-1}=0, S_{0}=s_{0} ; T\right) \tag{1}
\end{align*}
$$

As shown in Eq. 1, the collision probability at $T=t_{f}$ can be obtained by calculating the probability that no collision occurs for each time $T$ up until time $T=t_{f}$, which we refer to as the non-collision probability. The non-collision probability in Eq. 1 marginalizes over


Fig. 2 Schematic diagram for the framework of SRE.
several probability variables as follows:

$$
\begin{array}{rl}
\prod_{T=1}^{t_{f}} & p\left(C_{T}=0 \mid C_{T-1}=0, S_{0}=s_{0} ; T\right) \\
= & \int_{X_{E N V_{0}}} \int_{X_{O B J_{0}}} \sum_{V_{0}} \prod_{k=0}^{t_{f}-1}\left\{\int_{\Theta_{E N V_{k}}} \int_{\Theta_{O B k}} \int_{X_{O J_{k+1}}} \sum_{V_{k+1}}\right. \\
\quad & p\left(C_{k+1}=0, V_{k+1} \mid X_{O B J_{k+1}}, X_{E N V_{0}} ; C_{k}=0, V_{k}, \Theta_{O B J_{k}}, \Theta_{E N V_{k}}\right) \\
& p\left(\Theta_{E N V_{k}} \mid X_{O B J_{k+1}}, X_{O B J_{k}}, X_{E N V_{0}} ; C_{k}=0, V_{k}, \Theta_{O B J_{k}}\right) \partial \Theta_{E N V_{k}} \\
& p\left(X_{O B J_{k+1}} \mid X_{O B J_{k}}, X_{E N V_{0}} ; C_{k}=0, V_{k}, \Theta_{O B J_{k}}\right) \partial X_{O B J_{k+1}} \\
& \left.p\left(\Theta_{O B J_{k}} \mid X_{O B J_{k}}, X_{E N V_{0}} ; C_{k}=0, V_{k}\right) \partial \Theta_{O B J_{k}}\right\} \\
& p\left(X_{O B J_{0}}, V_{0} \mid X_{E N V_{0}} ; C_{0}=0, S_{0}=s_{0}\right) \partial X_{O B J_{0}} \\
& p\left(X_{E N V_{0}} \mid C_{0}=0, S_{0}=s_{0}\right) \partial X_{E N V_{0}}, \tag{2}
\end{array}
$$

where $X_{O B J_{k}}$ is a probability variable capturing the road user state at time $k$, which consists of position, velocity, and driver attributes, such as posture and age, and $X_{E N V_{0}}$ is a spatial representation of the current road environment including sidewalks and crosswalks. The probability variables $\Theta_{O B J_{k}}$ and $\Theta_{E N V_{k}}$ represent target factors and environment factors, respectively, at time $k$, each of which characterizes uncertainty in SBP. In addition, $V_{k}$ is a probability variable representing a binary state of reasonability concerning the predicted position of each road user at time $k$. The reasonability is subjectively judged based on the relationships between road users or between each road user and the road environment. When the driver judges the position of a road user at time $k$ is reasonable, the element of $V_{k}$ corresponding to that road user is equal to 0 , or otherwise is equal to 1 . In general, $V_{k}$ is defined as a multi-dimensional binary variable, the dimension of which depends on the number of road users and road segmentations.

$$
p\left(C_{k+1}=0, V_{k+1} \mid X_{O B J_{k+1}}, X_{E N V_{0}} ; C_{k}=0, V_{\mathrm{k}}, \Theta_{O B J_{k}}, \Theta_{E N V_{k}}\right)
$$

in Eq. 2 is a probability density function (PDF) corresponding to SRE in Fig. 2 and gives the probability that no collision occurs between the egovehicle and other road users at time $k+1$. Assuming that the state transition of each road user follows a Markov process, SPB in Fig. 2 is defined as $p\left(X_{O B J_{k+1}} \mid\right.$ $X_{O B J_{k}}, X_{E N V_{0}} ; C_{k}=0, V_{k}, \Theta_{O B J_{k}}$ ) in Eq. 2. The PDF describes the state of each road user at time $k+1$, which is determined by the previous state and the current spatial representation. Target factors and environment factors in Fig. 2 are represented as
$p\left(\Theta_{O J_{k}} \mid X_{O J_{k}}, X_{E N V_{0}} ; C_{k}=0, V_{k}\right)$ and $p\left(\Theta_{E N V_{k}} \mid X_{O B J_{k+1}}\right.$, $\left.X_{O B J_{k}}, X_{E N V_{0}} ; C_{k}=0, V_{k}, \Theta_{O B J_{k}}\right)$ in Eq. 2, respectively. The current states for each road user are defined as $p\left(X_{O B J_{0}}, V_{0} \mid X_{E N V_{0}} ; C_{0}=0, S_{0}=s_{0}\right)$ and the spatial representation of the current road environment is $p\left(X_{E N V_{0}} \mid C_{0}=0, S_{0},=s_{0}\right)$ in Eq. 2.

## 3. Algorithm for SRE and SBP

In order to simulate SRE, it is very important to select an appropriate algorithm using the subjective factors in the proposed framework. To develop the algorithm, we first simplify Eq. 2 to obtain Eq. 3, assuming conditional independence. We then assign each PDF some form of distribution function.

$$
\begin{aligned}
& \prod_{T=1}^{t_{f}^{\prime}} p\left(C_{T}=0 \mid C_{T-1}=0, S_{0}=s_{0} ; T\right)
\end{aligned}
$$

$$
\begin{align*}
& p\left(C_{k+1}=0 \mid V_{O B_{k+1}}\right) p\left(V_{\text {OB }_{k+1}} \mid X_{\text {OB }_{k+1}}\right) \\
& p\left(V_{E N V_{H+1}} \mid X_{O B B_{t+1}}, X_{E N N_{0}} ; \boldsymbol{\Theta}_{E N V_{0}}\right) \\
& \left.p\left(X_{O B_{k+1}} \mid X_{O B_{k}} ; \Theta_{O J_{0}}\right) \partial X_{O B_{k+1}}\right\} \\
& p\left(\Theta_{O J_{0}}\right) p\left(\Theta_{E N V_{0}}\right) \partial \Theta_{O B J_{0}} \partial \Theta_{E N V_{0}} \\
& p\left(X_{O B J_{0}} \mid X_{E N V_{0}} ; C_{0}=0, S_{0}=s_{0}\right) \partial X_{O B J_{0}} \\
& p\left(X_{E N V_{0}} \mid C_{0}=0, S_{0}=s_{0}\right) \partial X_{E N V_{0}} \tag{3}
\end{align*}
$$

Assuming that the ego-vehicle is on a horizontal plane, the PDFs of $X_{O B J_{0}}$ and $X_{E N V_{0}}$ in Eq. 3 are given as follows:

$$
\begin{align*}
& p\left(X_{O B J_{0}} \mid X_{E N V_{0}} ; C_{0}=0, S_{0}=s_{0}\right) \\
& =\sum_{n=1}^{N\left(s_{0}\right)} \frac{1}{N\left(s_{0}\right)} \mathcal{N}\left(\mu_{n, O B J_{0}}\left(s_{0}\right), \Sigma_{n, O B J_{0}}\left(s_{0}\right)\right),  \tag{4}\\
& p\left(X_{E N V_{0}} \mid C_{0}=0, S_{0}=s_{0}\right) \cong \delta\left(X_{E N V_{0}}-\mu_{E N V_{0}}\left(s_{0}\right)\right), \tag{5}
\end{align*}
$$

where $\mathcal{N}(\mu, \Sigma)$ in Eq. 4 is the normal distribution with mean $\mu$ and variance $\Sigma$. Thus, Eq. 4 represents a sum of normal distributions, $N\left(s_{0}\right)$ is the number of road users which includes the ego-vehicle, $X_{O B J_{k+1}}$ is a
probability variable of recognized state variables, mean, $\mu_{n, O B J_{0}}\left(s_{0}\right)$ and variance, $\Sigma_{n, O B J_{0}}\left(s_{0}\right)$ of a state of each road user depends on sensor inputs $s_{0}$. Then, the state variables consist of attribute $a_{n}$, position $\mathbf{x}_{n}$, and velocity $\mathbf{v}_{\mathrm{n}}$ : i.e., $\mathrm{a}_{\mathrm{n}}=$ \{ego-vehicle, pedestrian, other vehicle $\}, \mathbf{x}_{\mathrm{n}}=\left[\mathrm{x}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}}\right]^{\mathrm{T}}$, and $\mathbf{v}_{\mathrm{n}}=\left[\mathrm{v}_{\mathrm{n}, \mathrm{x}}, \mathrm{v}_{\mathrm{n}, \mathrm{y}}\right]^{\mathrm{T}}$. In Eq. 5, which has a discrete uniform distribution based on dividing the horizontal plane into M -dimensional grids, the M-dimensional state variable $X_{E N V_{0}}$ represents an attribute in each grid of the horizontal plane, such as $X_{E N V_{0}}=\left[\mathrm{e}_{1}, \mathrm{e}_{2}, \ldots, \mathrm{e}_{\mathrm{m}}, \ldots, \mathrm{e}_{\mathrm{M}}\right]$, and $\mathrm{e}_{\mathrm{m}}=\{$ curb, roadway, sidewalk, crosswalk $\}$, and $\delta(\cdot)$ is the Dirac delta function.
Here, $\Theta_{O B J_{0}}$ are the target factors and $\Theta_{E N V_{0}}$ are the environment factors, which indicate the magnitude of the constraints on the movements of each road user. Although various types of PDF can be used for $\Theta_{O B J_{0}}$ and $\Theta_{E N V_{0}}$, these variables are herein assumed to follow a discrete uniform distribution because the purpose of the present study is to confirm the suitability of the algorithm for simulating SRE using these variables. Under this assumption, PDFs take on the same form as Eq. 5 .

$$
\begin{align*}
& p\left(\Theta_{O B J_{0}}\right) \cong \delta\left(\Theta_{O B J_{0}}-\theta_{O B J_{0}}\right),  \tag{6}\\
& p\left(\Theta_{E N V_{0}}\right) \cong \delta\left(\Theta_{E N V_{0}}-\theta_{E N V_{0}}\right) . \tag{7}
\end{align*}
$$

For convenience, we refer to $\theta_{O B J_{0}}$ and $\theta_{E N V_{0}}$, respectively, as MVP and TPP, which are defined as follows:

Momentum variance parameter (MVP): the parameter relevant to the movement of a road user. When MVP becomes larger, the uncertainty in the movement of the road user becomes greater. Each road user has its own MVP depending on the uncertainty in its movement.

Transition possibility parameter (TPP): the parameter relevant to the constraint on the movement of a road user by the road environment. TPP regulates the movement of a road user at a location so that low TPP can inhibit the road user from moving, and high TPP can prompt road user to move.

SBP predominantly follows $p\left(X_{O B J_{k+1}} \mid X_{O B J_{k}} ; \Theta_{O B J_{0}}\right)$ in Eq. 3, which means that the state of each road user depends on the state at the previous time and the target
factors (MVP). In the present study, $X_{O B J_{k+1}}$ is assumed to follow the mixtures of normal distributions depending on each state of road users, $X_{n, O B J_{k}}$ and corresponding target factors $\Theta_{n, O B J_{0}}$.

$$
\begin{equation*}
X_{O B J_{k+1}} \sim \sum_{n=1}^{N\left(s_{0}\right)} \mathcal{N}\left(f_{n, O B J}\left(X_{n, O B J_{k}}\right), g_{n, O B J_{0}}\left(\Theta_{n, O B J_{0}}\right)\right) \tag{8}
\end{equation*}
$$

Although the target factors (MVP) are the dominant factors affecting SBP, TPP also affects SBP. Figure 3 is a conceptual illustration of SBP for a pedestrian using MVP and TPP. Figure 3(a) shows a traffic scene in which the ego-vehicle runs along the roadway and a pedestrian walks on the sidewalk parallel to the roadway. Figure 3(b) shows how SBP for the pedestrian is performed. Each circle in Fig. 3(b) indicates a probability distribution for the pedestrian's position at a given time. The pedestrian has MVP as its dynamics specification. The position distribution at a given future time can be obtained by iteratively updating using MVP and TPP for each location on the map. Over time, MVP spreads the position distribution while a low TPP restricts its spread. In Fig. 3(b), TPP is highest for the sidewalk area, the second highest for the crosswalk area, the third highest for the road area, and the lowest for the curb area. The distribution of the future position does not spread into the road because TPP is low for the curb and road areas, but does spread in the sidewalk and crosswalk areas because of high


Fig. 3 Example of SBP. (a) Traffic scene, and (b) SBP, which is predominantly affected by MVP. Subjective behavior prediction is also regulated by the TPP.

TPP in these areas.
However, for actual traffic scenes, it is difficult to represent the PDFs of the predictive variables $C_{k}$, $V_{O B J_{k+1}}, V_{E N V_{k+1}}$, and $X_{O B J_{k+1}}$ in Eq. 3 as continuous distribution functions because the variables suffer from nonlinear effects due to $\Theta_{O B J_{0}}$ and $\Theta_{E N V_{0}}$. In the present study, instead of using continuous distribution functions, we use the densities of numerous particles to form a flexible distribution.

The algorithm for SBP using a particle filter technique is described as follows:

1. Set the subjective parameters.
2. Allocate particles as current states on the horizontal plane according to Eq. 4.
3. Move particles according to random sampling using Eq. 8.
4. Remove particles with a probability proportionate to TPP at their positions (Fig. 4). Adding the number of removed particles and calculating an average, $p\left(V_{E N V_{k+1}}\right)$ can be approximated.
5. Remove particles if another object exists at the same location (Fig. 5). Adding the number of removed particles and calculating an average, $p\left(V_{O B J_{k+1}}\right)$ can be approximated.
6. Copy particles at a location with a probability that is inversely proportional to TPP.
7. Summarizing $V_{O B J_{k+1}}$, which describes the collision of particles with the ego-vehicle, $p\left(C_{k+1}=0\right)$ can be approximated.
Iterating steps 3 through 7 until $_{\mathrm{k}}=T$, the distribution of future positions can be generated. Although the removal processes shown in Figs. 4 and 5 are illustrated as separate, these processes can be performed in parallel, as described above. This algorithm guarantees that the total number of particles is constant over time by copying the same number of particles in step 6, as are removed in steps 4 and 5 . Due to this copying process, the computational cost is constant when the total number of particles remains constant, independent of the number of objects. The collision probability can be calculated by dividing the number of particles that are not removed by the number of particles that are removed.

## 4. Numerical Experiments

## 4. 1 Experimental Method

Figure 6(a) shows an example traffic scene used in the experiment. Figure 6(b) illustrates an expected
result for the collision probability, which shows the probability increasing monotonously over time.

Here, the horizontal plane, which is the region of interest, is $85.0[\mathrm{~m}]$ long (X-direction) and 20.0 [m] wide (Y-direction). The origin of the plane is set at the center of the ego-vehicle's position in the experiment. Dividing the region into small squares with side lengths of 0.25 [m] (27,200 squares in total). Then, the PDF of $X_{E N V_{0}}$ is described as follows:

$$
\begin{align*}
& p\left(X_{E N V_{0}} \mid C_{0}=0, S_{0}=s_{0}, T=t_{f}\right) \cong \delta\left(X_{E N V_{0}}-\mu_{E N V_{0}}\left(s_{0}\right)\right) \\
& \mu_{E N V_{0}}\left(s_{0}\right)=\left[e_{1}, e_{2}, \cdots, e_{27200}\right]^{T} . \tag{9}
\end{align*}
$$

In this experiment, it is assumed that $\Sigma_{n, O B J_{0}} \rightarrow 0$ in Eq. 4. Thus, the PDF of $X_{O B J_{0}}$ is described as follows:

$$
\begin{align*}
& p\left(X_{O B J_{0}} \mid X_{E N J_{0}} ; C_{0}=0, S_{0}=s_{0}, T=t_{f}\right) \\
& =\frac{1}{2}\left\{\delta\left(X_{E G O, O B J_{0}}-\mu_{E G O, O B J_{0}}\left(s_{0}\right)\right)+\delta\left(X_{P E D, O B J_{0}}-\mu_{P E D, O B J_{0}}\left(s_{0}\right)\right)\right\} \\
& \begin{aligned}
\mu_{E G O, O B J_{0}}\left(s_{0}\right) & =\left[x_{E G O, O B J_{0}}, y_{E G O, O B J_{0}}, v_{E G O, x, O B J_{0}}, v_{E G O, y, O B J_{0}}\right]^{T} \\
& =[0.0,0.0,10.0,0.0]^{T} \\
\mu_{P E D, O B J_{0}}\left(s_{0}\right) & =\left[x_{P E D, O B J_{0}}, y_{P E D, O B J_{0}}, v_{P E D, x, O B J_{0}}, v_{P E D, y, O B J_{0}}\right]^{T} \\
= & {\left[l_{E G O-P E D},-2.25,0.0,1.0\right]^{T} . }
\end{aligned}
\end{align*}
$$

The parameter $l_{E G O-P E D}$ is the depth distance between the ego-vehicle and the pedestrian.


Fig. 4 Prediction procedure on a horizontal plane modified by the TPP.


Fig. 5 Prediction procedure between different road users.

A prediction model is implemented as follows:

$$
\begin{align*}
\begin{aligned}
& \mid X_{O B J_{k}} \sim \lim _{\Gamma_{O B J_{E G O, 0} \rightarrow 0}} \\
& \mathcal{N}\left(f_{E G O}\left(X_{E G O, O B J_{k}}\right), g_{O B J_{E G O, 0}}\left(\Theta_{O B J_{0}}\right)\right) \\
& \times \mathcal{N}\left(f_{P E D}\left(X_{P E D, O B J_{k}}\right), g_{O B J_{P E D, 0}}\left(\Theta_{O B J_{0}}\right)\right) \\
& p\left(\Theta_{O B J_{0}}\right)=\delta\left(\Theta_{O B J_{0}}-\sigma_{P E D}^{2}\right) \\
& f_{E G O}\left(X_{E G O, O B J_{k}}\right)= {\left[x_{E G O, k}+\Delta t v_{E G O, x, k}, y_{E G O, k}\right.} \\
&\left.+\Delta t v_{E G O, y, k}, v_{E G O, x, k}, v_{E G O, y, k}\right]^{T} \\
&= {\left[x_{E G O, k}+10.0 \Delta t, y_{E G O k}, 10.0,0.0\right]^{T} }
\end{aligned} \\
\begin{aligned}
f_{P E D}\left(X_{P E D, O B J_{k}}\right)= & {\left[x_{P E D, k}+\Delta t v_{P E D, x, k}, y_{P E D, k}\right.} \\
& \left.+\Delta t v_{P E D, y, k}, v_{P E D, x, k}, v_{P E D, y, k}\right]^{T} \\
= & {\left[x_{P E D, k}, y_{P E D D, k}+\Delta t, 0.0,1.0\right]^{T} } \\
g_{O B J_{P E D, 0}}\left(\Theta_{O B J_{0}}\right)= & \operatorname{diag}\left[0.0,0.0, \sigma_{P E D}^{2}, \sigma_{P E D}^{2}\right] .
\end{aligned}
\end{align*}
$$

Here, $\Delta t$ is the time between $k$ and $k+1$. In this experiment, it is assumed that the ego-vehicle maintains uniform motion while the pedestrian walks randomly. In addition, $\sigma_{P E D}^{2}$ controls the subjective uncertainty of the pedestrian's movement as the expected variance from the ego-vehicle point of view.
The PDF of the subjective variable $\Theta_{E N V_{0}}$ is defined as follows:

$$
\begin{align*}
& p\left(\Theta_{E N V_{0}}\right)=\delta\left(\Theta_{E N V_{0}}-\theta_{\text {ENV }}\right) \\
& \theta_{\text {ENV }}= {\left[\alpha_{\text {EGO-CURB }}, \alpha_{\text {EGO-ROADWAY }}, \alpha_{\text {EGO-SIDEWALK }}, \alpha_{\text {EGO-CROSSWALK }},\right.} \\
&\left.\alpha_{\text {PED-CURB }}, \alpha_{\text {PED-ROADWAY }}, \alpha_{\text {PED-SIDEWALK }}, \alpha_{\text {PED-CROSSWALK }}\right]^{T} \\
&=\left[0.0,1.0,0.0,1.0,0.0, \alpha_{\text {PED-ROADWAY }}, 1.0, \alpha_{\text {PED-CROSSWALK }}\right]^{T} . \tag{12}
\end{align*}
$$



Fig. 6 Settings used in the numerical experiments.

The likelihood $\alpha_{X-Y}$ is the subjective validity of the proposition "An object X can move to place Y."

## 4. 2 Results

## 4. 2. 1 Effect of Target Factors on SBP

As described in Section 3, the target factors that the driver subjectively perceives in a traffic situation are represented by MVP in the proposed algorithm. For example, when the driver sees a pedestrian looking around, the driver predicts the behavior of the pedestrian with large uncertainty. Thus, the effect of MVP on collision probability, which is calculated using behavior prediction, is confirmed in this experiment.

Figure 7 shows the mean collision probabilities at $T=4.0$ [s] over 100 trials as a function of $l_{\text {EGO-PED }}$ and $\sigma_{\text {PED }}^{2}$ for $\alpha_{\text {PED-ROADWAY }}=\alpha_{\text {PED-SIDEWALK }}=1.0$.
When $\sigma_{P E D}^{2}=0.0^{2}\left[\mathrm{~m}^{2} / \mathrm{s}^{2}\right]$ (a pedestrian is not aware of the ego-vehicle), the collision probabilities are almost the same as the analytic results calculated by the geometric relation between the ego-vehicle and the pedestrian. In contrast, when $\sigma_{P E D}^{2}>0.0^{2}\left[\mathrm{~m}^{2} / \mathrm{s}^{2}\right]$ (a driver perceives uncertainty in the pedestrian's locomotion), the results in Fig. 7 become broader.

## 4. 2. 2 Effect of Environment Factors on SBP

As described in Section 3, environment factors that the driver subjectively perceives in a traffic scene are represented as TPP values in the proposed algorithm. For example, barriers separating the sidewalk from the roadway decrease the subjective uncertainty in the


Fig. 7 Effect of MVP on SBP. A large MVP $\left(\sigma_{P E D}^{2}\right)$ raises collision probability against the pedestrian at the far distance because it adds an uncertainty to future position of the pedestrian.
pedestrian's behavior. In particular, the driver thinks that the pedestrian is unlikely to cross the roadway (the pedestrian can be expected to walk along the sidewalk). In this experiment, it is confirmed that adjusting TPP can result in a reasonable collision probability.
Figure 8 shows the mean collision probabilities at $T$ $=4.0[\mathrm{~s}]$ over 100 trials as a function of $l_{E G O-P E D}$ and for condition of $\sigma_{P E D}^{2}=0.1^{2}\left[\mathrm{~m}^{2} / \mathrm{s}^{2}\right]$ and $\alpha_{\text {PED-CROSSWALK }}$ $=1.0$. As the TPP of the roadway, $\alpha_{\text {PED-ROADWAY }}$, becomes low, the collision probability becomes low, which implies that the pedestrian is less likely to enter the roadway in such a traffic environment.

## 4. 2. 3 Effect of the Presence of a Crosswalk on SBP

Let us consider a realistic traffic scene in which a pedestrian is walking toward a crosswalk. In the proposed algorithm, the crosswalk is regarded as an environment factor, which means that TPP is high for the crosswalk for both the ego-vehicle and pedestrians. The geometric setting is shown in Fig. 9, which is the same that in Fig. 6(a) except for the addition of a crosswalk. Numerical experiments were conducted by varying TPP in the area of the roadway for the pedestrian.
Figure 10 shows the mean collision probabilities at time $T=4.0$ [s] over 100 trials as a function of $l_{E G O-P E D}$ and $\alpha_{\text {PED-CROSSWALK }}$ for $\sigma_{P E D}^{2}=0.1^{2}\left[\mathrm{~m}^{2} / \mathrm{s}^{2}\right]$ and $\alpha_{\text {PED-ROADWAY }}=0.0$. When TPP is greater than or equal to 0.50 , the collision probabilities are almost the same as those in Fig. 8. This result shows that the pedestrian is likely to cross at areas other than the crosswalk. In


Fig. 8 Effect of the TPP on SBP. A small TTP ( $\alpha_{\text {PED }}$ ROADWAY) decreases overall collision probability against the pedestrian because it is regulates movement of the pedestrian.
contrast, when TPP is equal to 0.0 , the collision probability has a peak near the crosswalk (depth distance: 20.0 [m]). This means that a pedestrian who is very close to the crosswalk is predicted to cross at the crosswalk. If TPP is instead equal to 0.25 , the collision probability is slightly broader than in the case that TPP is equal to 0.0 . This means that the pedestrian near the crosswalk is predicted to cross the road. Thus, the algorithm can simulate the performance of SBP of a human driver by adapting TPP.


Fig. 9 Geometric setting for a numerical experiment to evaluate the effect of the presence of a crosswalk on SBP. The TPP of the crosswalk for the pedestrian is equal to that of the sidewalk, the value of which is generally set to a lower value than that of the roadway.


Fig. 10 Effect of the presence of a crosswalk on SBP. Relationship between collision probability and a pedestrian walking near a crosswalk ( $l_{\text {EGO-PED }}=$ 20.0 [m]) remains high because TPP near the crosswalk is high, which prompts the pedestrian to walk along the crosswalk.

## 5. Conclusion

In the present paper, a framework for calculating SBP and SRE for a traffic situation has been proposed. Interviewing expert drivers to obtain the cognitive structure of the risk estimation process, subjective factors, which play an important role in SBP and SRE, have been classified into target factors and environment factors. These factors were modeled as probability variables in a probability distribution of collision probability following a Bayesian approach. Based on the mathematical framework, the collision probability can simulate the magnitude of subjective risk by assuming appropriate PDFs for the probability variables. Although various distribution types for these variables are possible, a simple distribution, i.e., a uniform distribution, was adopted in order to confirm the suitability of the proposed algorithm. These distributions were parameterized using only two parameters, namely, MVP and TPP. A particle filter was used to generate SBP in order to estimate the collision probability. The results of numerical experiments verify that the proposed algorithm can estimate a suitable collision probability with only two parameters in a simple distribution.

In the future, the proposed algorithm will be examined for less orderly road environments. Moreover, the selection of appropriate distributions for the subjective factors for useful applications will be considered.

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Figs. 1 and 3-10
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