



Research
Report

Topology Optimization of Electromagnetic Materials

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Abstract

This paper describes a topology optimization method that has been designed for application to the structures of electromagnetic materials. The goal of topology optimization is to produce high-performance structures. As such, it has been extensively applied to a variety of structural optimization problems. Also, its application to the task of electromagnetic material structure design has the potential to be extremely useful. In this article, we introduce two topology optimization methods for the structural design of

electromagnetic materials. One is based on the periodic boundary Finite Element Method (FEM) that is used to design periodic material structures. The second method is based on time domain analysis using the Finite Difference-Time Domain (FDTD) method where we can directly deal with the frequency characteristics that are formulated as an integral through a continuous frequency range suitable for RF device design. Several design examples are presented in order to confirm the usefulness of the proposed method.

Keywords

Topology optimization, Electric magnetic field, Dielectric material, Finite element method, Finite difference time domain method

1. Introduction

Topology optimization is the most flexible optimization method that can simultaneously deal with geometrical and topological configuration changes.¹⁾ This method involves defining a fixed design domain such that it is larger than the resulting design. In the fixed domain, an arbitrary configuration can be expressed using a characteristic function, allowing large changes in the geometrical and topological design during the optimization process. Topology optimization was originally developed for structural design, and recently adapted for many other areas of design by taking other branches of science into consideration, such as electromagnetics.²⁾ These efforts have focused on electrostatics,³⁾ and static magnetics.⁴⁾ On the other hand, topology optimization has been applied to electromagnetic wave propagation problems. Photonic crystal wave guides have been designed by considering wave propagation problems in optical waveguides using the Finite Element Method (FEM).^{5, 6)} Furthermore, Kiziltas et al. developed an optimum design method for patch antennas using the Finite Element-Boundary Integration method (FE-BI)⁷⁾

This article introduces two topology optimization methods for the structural design of electromagnetic materials. One is based on periodic boundary FEM for the design of a periodic material structure. The other is based on time domain analysis using the Finite Difference-Time Domain (FDTD) method, which allows us to directly handle the frequency characteristic that is formulated as an integral through a continuous frequency range suitable for RF device design. First, the concept of topology optimization is briefly discussed. Next, a new objective function and optimization problem are formulated and, based on these formulations, an optimization algorithm is constructed. Finally, several design examples are presented to confirm the usefulness of the proposed method.

2. Optimization method

2.1 Concept of topology optimization

In this section, we briefly discuss the concept of topology optimization. Consider the problem of

determining the boundary of a design domain Ω_d by minimizing or maximizing an objective function. The key idea of topology optimization is the introduction of a fixed, extended design domain D that includes the original design domain Ω_d , a priori, and which utilizes the following characteristic function χ_Ω :

$$\chi_\Omega(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{x} \in \Omega_d \\ 0 & \text{if } \mathbf{x} \in D \setminus \Omega_d \end{cases} \dots\dots\dots (1)$$

where \mathbf{x} denotes a position in the extended design domain D .

Topology optimization is defined as the problem of finding the optimal distribution of $\chi_\Omega(\mathbf{x})$, where an objective function is optimized, subject to constraint specifications including governing physical equations. The numerical treatment of $\chi_\Omega(\mathbf{x})$ is, however, very difficult because $\chi_\Omega(\mathbf{x})$ may have an infinite number of discontinuous binary values at every point in D . To avoid this problem, $\chi_\Omega(\mathbf{x})$ must be relaxed and expressed as an appropriate continuous function. Several relaxation methods have been proposed, such as a homogenization method¹⁾ and density method, which is also known as the SIMP method.⁸⁾ When the homogenization method is used, a microstructure for representing the composite material in the design domain is introduced first. A continuous relaxed function representing the material distribution in a global sense is obtained by calculating the homogenized properties. On the other hand, the SIMP method uses a fictitious isotropic material whose property tensor is assumed to be a function of the penalized material density, expressed by an exponent parameter when this method is applied to structural problems. The SIMP method is widely used and has been applied to a variety of design problems, due to the simplicity of its formulation and implementation.

In this study, we apply a density method to the design of an antenna. Here, we assume that the design domain is composed of dielectric materials, and that the physical property determining the antenna's performance is electric permittivity. The relative permittivity at point \mathbf{x} can be expressed as:

$$\epsilon_\gamma(\rho(\mathbf{x})) = \epsilon_\gamma^{\text{solid}} + (\epsilon_\gamma^{\text{solid}} - \epsilon_\gamma^{\text{air}})\rho(\mathbf{x}), (0 \leq \rho(\mathbf{x}) \leq 1) \dots\dots\dots (2)$$

where ϵ_r^{air} is the relative permittivity in air, and $\epsilon_r^{\text{solid}}$ is that in a solid. Here we use a simple linear interpolation for the density function, since the permittivity changes linearly with respect to the normalized density $\rho(x)$.

2.2 Periodic material structure design

A rectangular domain with the incidence boundary at the upper side and the output boundary at the lower side is prepared for analysis (Fig. 1). The domain is bounded by the periodic boundary to the right and left, and has an infinite cycle in the x direction and a finite length in the y direction. The dimensions of the boundary are d in the x direction and $5d$ in the y direction, where $d = \lambda_0$ in the model. Moreover, the variables at the node shared through the periodic boundary are constrained with phase differences to express the incident angle θ .

Next, the objective function is formulated so that the power flowing out through a design domain can be minimized across the entire design domain. The power flowing out through the design domain is described as follows:

$$P_t = \iint_{\Omega_t} \Re \left[-\frac{1}{2} E_z \left(-\frac{1}{j\omega\mu_0} \frac{\partial E_z}{\partial y} \right)^t \right] dx dy \dots\dots (3)$$

where Ω_t , ω and μ_0 denote the evaluation domain, radian frequency and permeability in air, respectively (Fig. 2). Then, $|S_{21}^{\text{area}}|$ is calculated as

$$|S_{21}^{\text{area}}| = \frac{P_t / A_t}{P_{\text{inc}} / d} \dots\dots\dots (4)$$

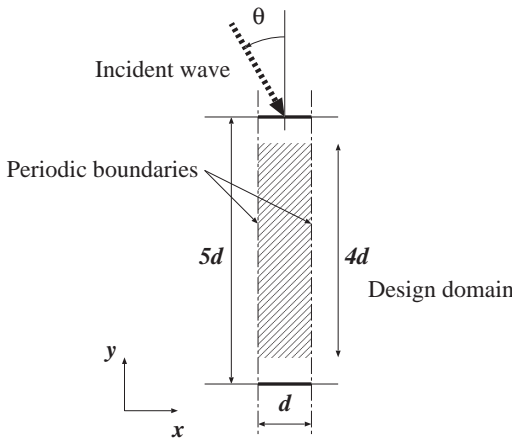


Fig. 1 Model for periodic FEM analysis.

where A_t is the area of the evaluation domain, P_{inc} is the incident power, and d is the length of the incident side.

2.3 Material structure design of RF components

Electromagnetic topology optimization can also be applied to the material structure design of RF components by setting the analysis domain to open space. Here, we demonstrate its capability by designing dielectric resonator antennas (DRA).

Figure 3 shows the analysis model for an antenna system, used for the design of a DRA. As shown in this figure, the analysis domain consists of a rectangular parallelepiped volume including the extended design domain D that is placed on the upper side of the metallic ground plane. The feed probe, a short length of bare wire protruding into the design domain, transmits electromagnetic waves from the shielded coaxial cable connecting the design domain to the electronic circuit driving the

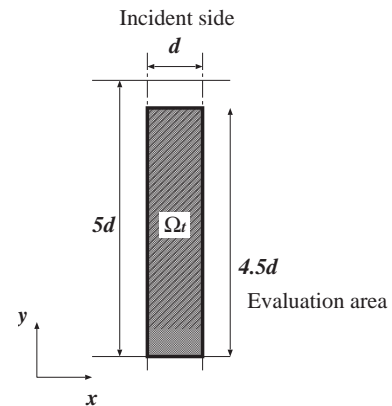


Fig. 2 Evaluation domain.

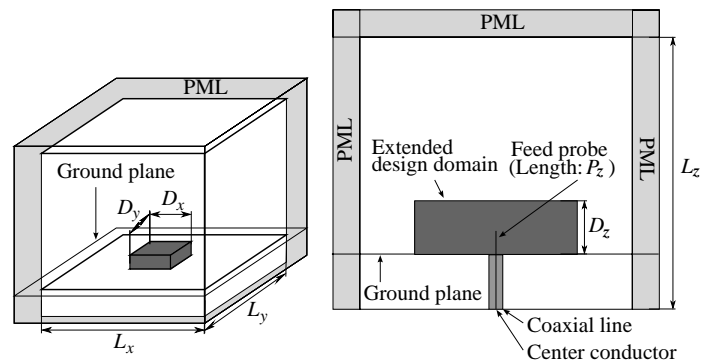


Fig. 3 DRA analysis model.

antenna, and electromagnetic waves radiated from the design domain are then analyzed in the analysis domain that includes it. Perfectly Matched Layers (PML)⁹ that absorb electromagnetic waves cover five faces of the analysis domain, while the bottom surface is left bare.

Here, the reflection coefficient $|S_{11}(\omega)|$, i.e. return loss, as formulated by the following:

$$|S_{11}(\omega)| = \left| \frac{P_{\text{ref}}(\omega)}{P_{\text{in}}(\omega)} \right| \dots\dots\dots (5)$$

is a measure of the radiation efficiency of the electromagnetic waves radiating from the design domain where $P_{\text{in}}(\omega)$ is the input power and $P_{\text{ref}}(\omega)$ is the reflected power in the frequency domain. That is, by minimizing $|S_{11}(\omega)|$, the radiated power $P_{\text{rad}}(\omega)$, which stands for the wave radiation performance of the antenna, is maximized since we are operating under the lossless assumption formulated as

$$P_{\text{in}}(\omega) = P_{\text{rad}}(\omega) + P_{\text{ref}}(\omega) \dots\dots\dots (6)$$

Note that, Eq. (5) is most commonly used as a performance measure for antenna development.

Since $P_{\text{in}}(\omega)$ is constant in this optimization problem, minimizing $|S_{11}(\omega)|$ is equivalent to minimizing $P_{\text{ref}}(\omega)$ and when the input pulse has a power spectrum with the same peak frequency as the antenna's working frequency, and has the required bandwidth, minimizing $P_{\text{ref}}(t)$ subject to the chosen input pulse minimizes $P_{\text{ref}}(\omega)$ in the specified bandwidth, with the input pulse weighted to have its peak at the working frequency. Thus, the objective function is formulated as an integral value of $P_{\text{ref}}(t)$ with respect to time, as follows:

$$F = \int_0^T P_{\text{ref}}(t) dt \dots\dots\dots (7)$$

where T is a fixed final time. Also, a Gaussian pulse having a center frequency equal to the target frequency, and a -3 dB power bandwidth as the specified bandwidth, is selected for the input pulse. Therefore, the optimization problem is formulated as:

$$\text{Minimize } \int_0^T P_{\text{ref}}(t) dt \dots\dots\dots (8)$$

subject to: Maxwell's equation
 $0 \leq \rho \leq 1$

3. Numerical implementation

3.1 Optimization algorithm

Figure 4 is a flowchart of the optimization procedure. First, we set initial data for optimization and analysis. For the initial configuration, the design domain D has a uniform density distribution such that $\rho(\mathbf{x}) = 0.5, (\mathbf{x} \in D)$. During the optimization procedure, in the first step, FDTD analysis is performed and the time history of the electric field is obtained. Then, the objective function in Eq. (7) is calculated using the obtained field values.

Design variables are placed at the center of each Yee cell, which is bounded by electric field components along its edges. For all cells in D , we define design variable p_i as

$$p_i = \rho(\mathbf{x}_i) \dots\dots\dots (9)$$

where \mathbf{x}_i is the center of the i th cell in fixed design domain D .

If the objective function has converged, the procedure finishes. Otherwise, the FDTD analysis is executed to obtain adjoint variable values that are used in subsequent analysis to calculate the design sensitivities. In the last step, the design variables are updated using Sequential Linear Programming (SLP) and the procedure returns to the first step for the next iteration.

4. Numerical examples

4.1 Periodic material structure design

This example shows whether it is possible to construct a topology optimization method to make

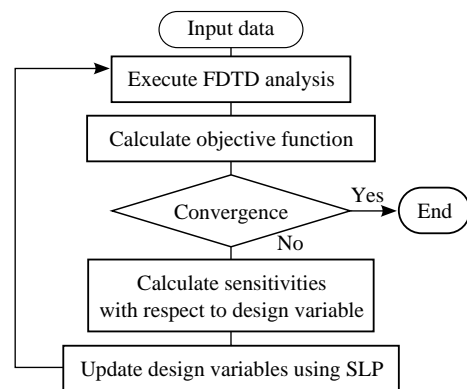


Fig. 4 Flowchart of optimization procedure.

predictions related to the specific actions that a multi-dimensional periodic structure may take in response to electromagnetic waves. As an example, we consider a 2-dimensional electromagnetic band gap structure consisting of a dielectric and air.

The optimization problem is formulated as:

$$\text{Minimize } \sum_{\rho} w_i |S_{21}^{\text{area}}|_{f_i}^{\theta_i} \dots\dots\dots (10)$$

subject to: Helmholtz equation

$$0 \leq \rho \leq 1$$

$$V_f \geq 0.1$$

where V_f indicates the volume fraction of the dielectric material, and where θ_i is set to 0° and 45° , respectively. Also, w_i is the weighting parameter which is set to the same value.

Figure 5 shows the iteration history for a 2-dimensional structure and its electric amplitude distribution for multiple loaded inclined incident waves after 20 iterations. These conditions allowed us to capture the generation of a 2-dimensional electromagnetic band gap structure.

4. 2 Material structure design of RF components

We designed the structure of the wideband DRA in order to confirm the usefulness of the proposed method. Figure 3 shows the analytical model that we used for the rectangular DRA mounted on the ground plane. The DRA with dimension $D_x = 30$ mm, $D_y = 30$ mm, $D_z = 8$ mm and relative dielectric constant $\epsilon_r^{\text{solid}} = 12.0$ is fed at the center by the coaxial probe that extends $P_z = 5$ mm into the DRA. We chose the calculated area with 216,000 ($60 \times 60 \times 60$) cubic cells of 1 mm. A Gaussian pulse with a center frequency of 5.0 GHz and a half power bandwidth of 1.0 GHz were selected as the input pulse, therefore, the working bandwidth of the designed antenna will be 4.5 GHz to 5.5 GHz.

Figure 6 shows the iteration history of the bottom view of the designed material structure reflection coefficient $|S_{11}|$ for the initial, 10th, 20th and final configuration. The obtained configuration has a gradient air gap and an arch shape concentric to the probe. The achieved return loss in the working range was less than -20 dB.

5. Conclusion

We proposed two topology optimization methods for the structural design of electromagnetic

materials.

A topology optimization method for periodic electromagnetic material structures was introduced. It is based on periodic boundary FEM. Physically appropriate periodic structures were obtained by setting the objective function to the propagation power across the entire design domain.

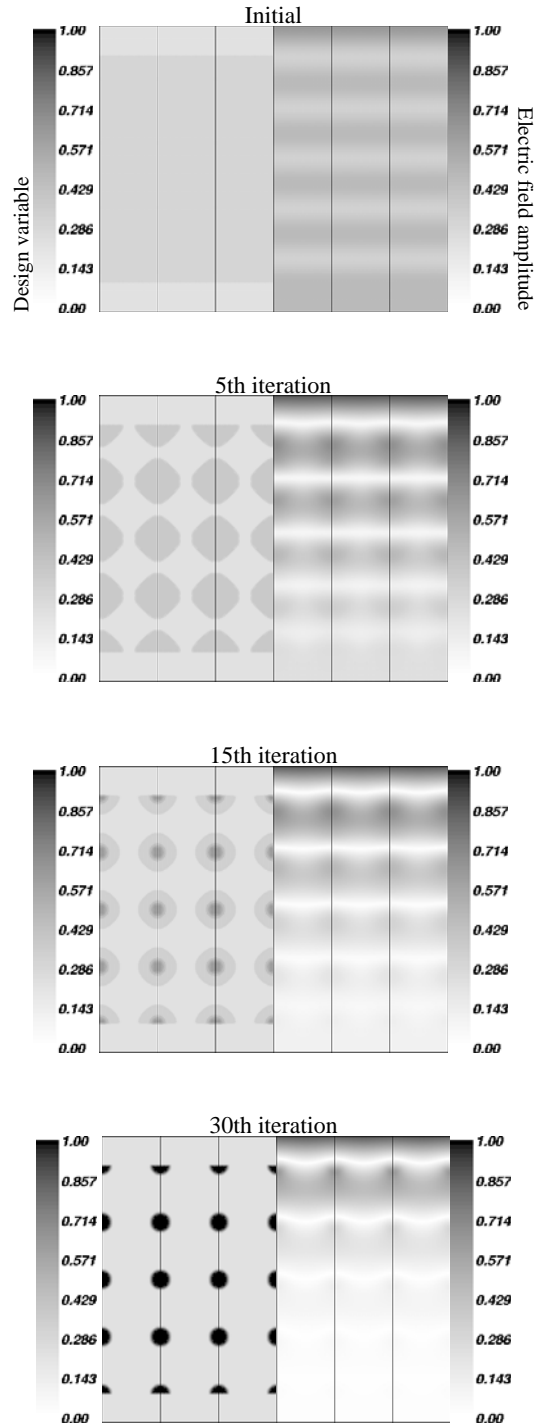


Fig. 5 Optimal configuration and electric field distribution.

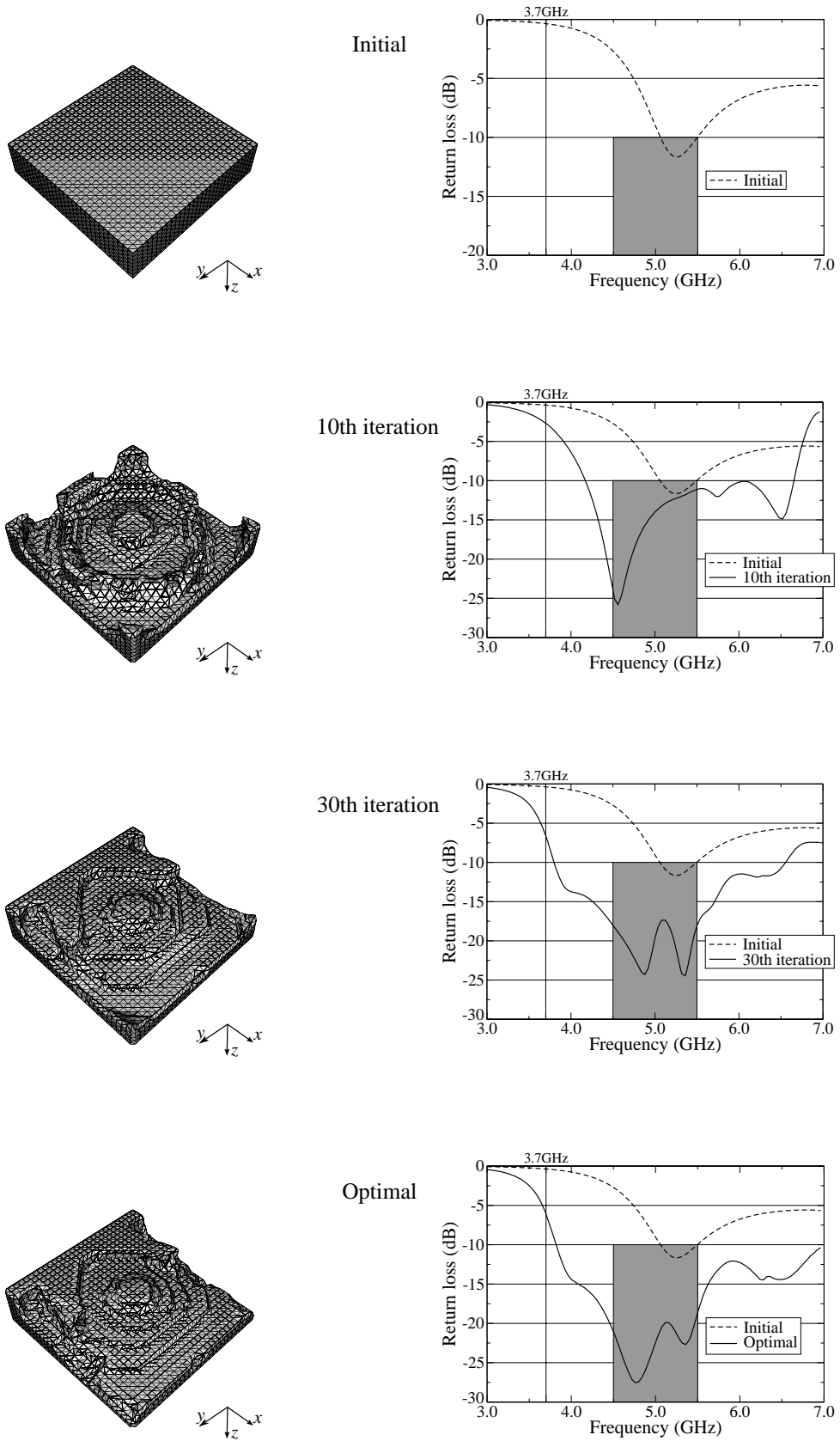


Fig. 6 Iteration history of bottom views of design configuration and return losses.

The other is a time-domain topology optimization method for designing the optimal material structure of RF components using the FDTD method. A DRA design example was provided to examine the usefulness of the proposed method. We confirmed that the proposed method enables broadband dielectric resonator antenna design simply by specifying the operational frequency range by using the peak frequency and bandwidth of a Gaussian pulse.

References

- 1) Bendsøe, M. P. and Kikuchi, N. : "Generating Optimal Topologies in Structural Design Using a Homogenization Method", *Comput. Methods in Appl. Mech. and Eng.*, **71**(1988), 197-224
- 2) Dyck, D. N. and Lowther, D. A. : "Automated Design of Magnetic Devices by Optimizing Material Distribution", *IEEE Trans. Magnetics*, **32-3**(1996), 1188-1193
- 3) Byun, J. K., Park, I. H. and Hahn, S. Y. : "Topology Optimization of Electrostatic Actuator Using Design Sensitivity", *IEEE Trans. on Magnetics*, **38-2**(2002), 1053-1056
- 4) Yoo, J., Kikuchi, N. and Volakis, J. L. : "Structural Optimization in Magnetic Devices by the Homogenization Design Method", *IEEE Trans. on Magnetics*, **36-3**(2000), 574-580
- 5) Jensen, J. S. and Sigmund, O. : "Systematic Design of Photonic Crystal Structures Using Topology Optimization: Low-loss Waveguide Bends", *Appl. Phys. Lett.*, **84-12**(2004), 2022-2024
- 6) Tsuji, Y., Hirayama, K., Nomura, T., Sato, K. and Nishiwaki, S. : "Design of Optical Circuit Devices Based on Topology Optimization", *IEEE Photonics Technol. Lett.*, **18-7**(2006), 850-852
- 7) Kiziltas, G., Psychoudakis, D., Volakis, J. L. and Kikuchi, N. : "Topology Design Optimization of Dielectric Substrates for Bandwidth Improvement of a Patch Antenna", *IEEE Trans. on Antennas and Propag.*, **51-10**(2003), 2732-2743
- 8) Bendsøe, M. P. : "Optimal Shape Design as a Material Distribution Problem", *Struct. Multidisc. Optim.*, **1-4**(1989), 193-202
- 9) Berenger, J. : "A perfectly Matched Layer for the Absorption of Electromagnetic Waves", *J. Computat. Phys.*, **114**(1994), 185-200

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