Research Report

Design of Optical Devices Based on Topology Optimization

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Abstract

We have developed a CAE technique for designing metamaterials, which offer superior electromagnetic properties due to their internal structures being of a size that is less than the wavelength of incident waves. In our CAE technique, the internal structures of metamaterials are designed using a topology optimization method. Optical devices are key components in the search for energy-conserving or high-performance automobiles. Since optical devices handle electromagnetic waves such as visible light, the efficiency of the devices is significantly influenced by their internal structures, in exactly the same way as with metamaterials. So, we have examined the applicability of the developed CAE technique to the design of the internal optical structures of optical devices. Numerical simulations of the dielectric band gap structure and optical waveguides were performed as part of the verification. From the results obtained for the dielectric band gap structure, multilayer structures perpendicular to the direction of the incident waves, which we would expect to obtain as a result of simulating an electromagnetic band gap structure, were obtained for several incident angles. Moreover, because the structures for propagating input power efficiently are designed for 90-degree and T-branch waveguides, the validity of the developed CAE technique was confirmed.

Keywords: Optical device, Topology optimization, Dielectric band gap structure, Optical waveguide, Numerical simulation

1. Introduction

Optical devices such as light-emitting diodes (LEDs), solar cells and optical waveguides hold the promise of enabling energy-conserving or high-performance automobiles. Currently, however, the efficiency of such optical devices is insufficient. For LEDs and solar cells, the materials used for the devices have been improved to enhance the device efficiency. In addition to materials improvement, there are some other methods that can be used to enhance efficiency. One involves modifying the internal structures of the device and/or the device surfaces. Because the light utilized for optical devices, which is either visible light or near-infrared light, is a kind of electromagnetic wave, the light behaves according to the properties of wave motion, and so the propagation of light in the devices is greatly affected by the structure in the light propagating region and/or the boundary between the materials in the devices. To enhance a device's efficiency, therefore, it is useful to improve the device's internal structure.

We have developed a computer aided engineering (CAE) technique for designing metamaterials,[1-3] in which the internal structures provide superior electromagnetic wave properties. With this technique, the internal structures of the metamaterials are designed using a topology optimization method,[4-7] which was originally developed for structural optimization problems. We believe that the developed technique can be applied to other structural designs that involve electromagnetic waves.

In this paper, we examine the applicability of the developed CAE technique to the design of the internal structures of optical devices. First, the topology optimization method is briefly introduced, after which the numerical results for the dielectric band gap structure[8] and optical waveguides[9] are considered.

2. Topology optimization

Topology optimization is an optimization method that can simultaneously deal with not only geometric forms but also topological configurations. This method does not require the boundary for representing the shape of a structure because the shape is expressed by the density distribution in the design domain instead of the boundaries. The design domain is discretized using finite elements, each of which has density that acts as a design variable. The values of the objective function and design sensitivities are also defined for each element, and are calculated numerically. Using the evaluated values and sensitivities of the objective function, the design variables are updated so that an objective function is minimized when using nonlinear programming, thus improving the density distribution in the design domain to obtain an optimal configuration.

The numerical methods used in this study are as follows:

1) The electromagnetic field in the analysis domain is calculated using a finite element method.

2) The adjoint variable method is used to calculate the sensitivities.

3) Sequential linear programming is employed for the optimization calculations.

The density of each element is associated with a dielectric constant for one using the following equation.

$$\varepsilon_i = \varepsilon_{\min} + (\varepsilon_{\max} - \varepsilon_{\min})\rho_i^p \qquad (0 \le \rho \le 1) \quad \cdots\cdots\cdots (1)$$

where i indicates the element number, and $\varepsilon_{\min}$ and $\varepsilon_{\max}$ are the minimum and maximum relative permittivity utilized for the design, respectively. ρ is the density, and p is the penalization parameter that promotes convergence away from the grayscale results.

3. Numerical results for dielectric band gap structure

The developed CAE technique was applied to a dielectric band gap structure in order to verify its effectiveness for optical devices. **Figure 1** shows the FEM model used for this examination. The upper and lower sides have incident and output boundaries, respectively, and the periodic boundaries are specified on both the right and left. The size of the design domain is $4d$ where $d = 0.35\lambda_0$; the wavelength in free space. In this test, we assume $\varepsilon_{\min} = 1.0$, $\varepsilon_{\max} = 10.5$, and $p = 0.5$.

3. 1 Perpendicular incident wave

If incident waves enter the design domain perpendicularly, a one-dimensional multilayer structure should arise, as shown in **Fig. 2**. To verify this, a design optimization is performed for several frequencies while maintaining the incident angle θ at 0°. The objective function is specified as the minimization of the amplitude of the transmission coefficient $|S_{21}|$ that is obtained from the incident waves and the electric field of the output terminal, as follows:

$$S_{21} = \{g\}^t \{E_{out}\} \cdots\cdots\cdots\cdots\cdots (2)$$

where $\{g\}$ is vector constant calculated from the incident wave, $\{E_{out}\}$ is the electric field vector along the output terminal and $\{\}^t$ denotes the conjugate transpose.

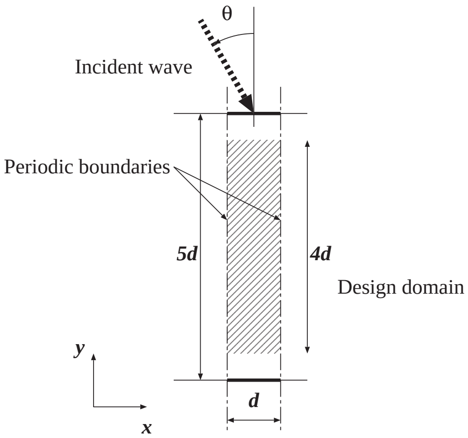


Fig. 1 Model for FEM analysis.

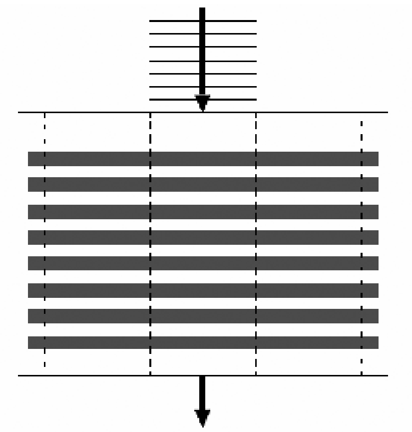


Fig. 2 Prospective structure for perpendicular incident waves.

Figure 3 shows the optimal structure calculated. We see that a multilayer structure, as would be expected for a perpendicular incident wave, was constructed. Moreover, a multilayer structure was also obtained for other frequencies.

3. 2 Inclined incident wave

In this section, we examine the optimal structures for inclined incident waves. The objective function is formulated by using the amplitude of the transmission coefficient $|S_{21}|$, in the same as in the above section. **Figure 4** shows the calculated results for an incident angle of 15°. Although we would assume that we would obtain a multilayer band gap structure inclined at 15° with respect to the vertical axis, the optimized structure shown in Fig. 4 is actually different.

To overcome this problem, the objective function

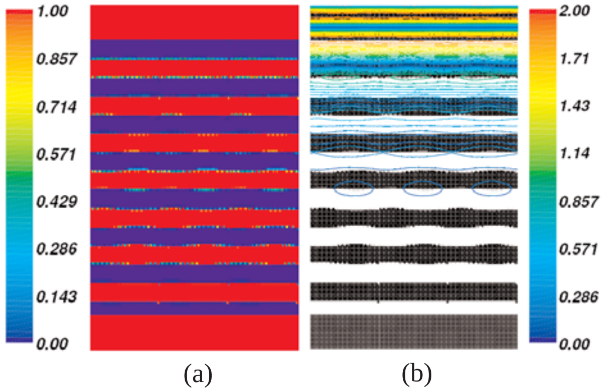


Fig. 3 Optimal structure calculated for perpendicular incident wave. (a) density distributions as design variables, (b) amplitude of electric field.

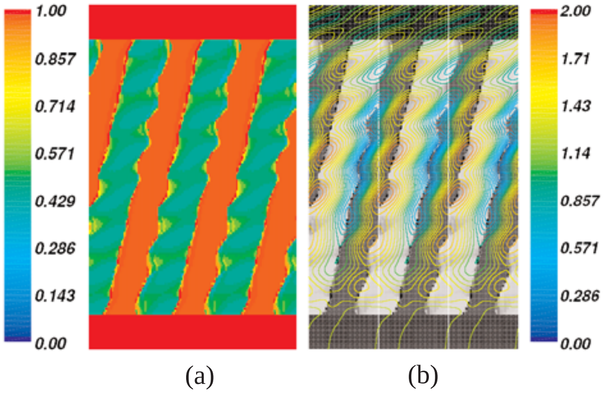


Fig. 4 Optimal structure calculated using Eq. 2 for incident angle 15°. (a) density distributions as design variables, (b) amplitude of electric field.

was modified so that the power flowing out through the design domain is minimized across the entire design domain. Thus, the power flowing out through the design domain is described as follows:

$$P_t = \iint_{\Omega_t} \Re\left[-\frac{1}{2}E_z\left(-\frac{1}{j\omega\mu_0}\frac{\partial E_z}{\partial y}\right)^t\right] dxdy \cdots\cdots (3)$$

where Ω_t, ω and μ_0 denote the evaluation domain, angular frequency and permeability in free space, respectively. Then, $|S_{21}|$ is calculated as

$$S_{21} = \frac{P_t / A_t}{P_{inc} / d} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (4)$$

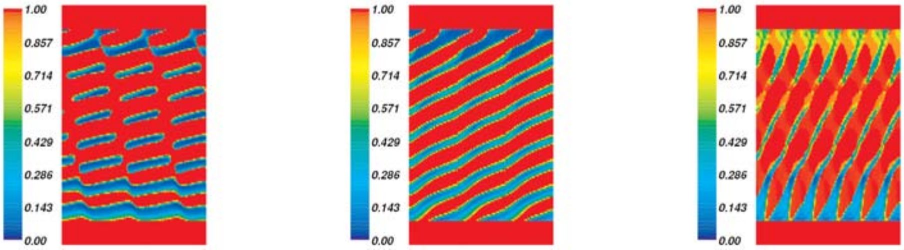
where A_t, P_{inc} and d are the area of the evaluation domain, the incident power and the length of the incident side, respectively. **Figure 5** shows the optimal structures obtained for incident wave angles of 15^o, 30^o and 60^o, respectively. We can see that all the results exhibit a multilayer band gap structure perpendicular to each incident direction, although slight disrupt patterns appear in the optimized structures.

4. Numerical results for optical waveguides

The developed CAE technique was applied to the design of two kinds of optical waveguides, namely, a 90-degree bend and a T-branch. **Figure 6** shows the configuration of the 90-degree bend model. The waveguide width is w = 0.7 μm, the design region size is W = 4 μm, and d = 1.5 μm. The refractive indexes used for the topology optimization are between 1.0 and 1.45. A fundamental TE mode with wavelength λ = 1.55 μm is assumed as an incident wave. The objective function is specified as follows:

$$Maximize\,|S_{21}|^2 \text{ at } \lambda = 1.55\ \mu\text{m} \cdots\cdots\cdots\cdots (5)$$

where S_{21} is the ratio of the output amplitude to the input one. **Figure 7** shows the calculated refractive index profile and propagating field intensity. In both Fig. 7 (a) and (b), the darker areas represent distributions with relatively large values. From



(a) (b) (c)

Fig. 5 Optimal structures calculated using Eqs. 3-4 for inclined incident waves. Incident angles are 15^o, 30^o and 60^o for (a), (b) and (c), respectively.

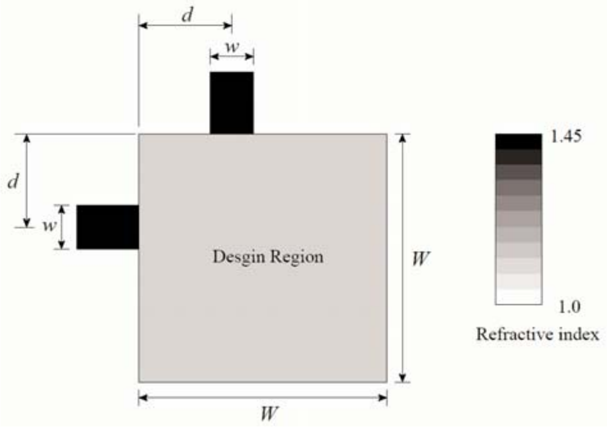
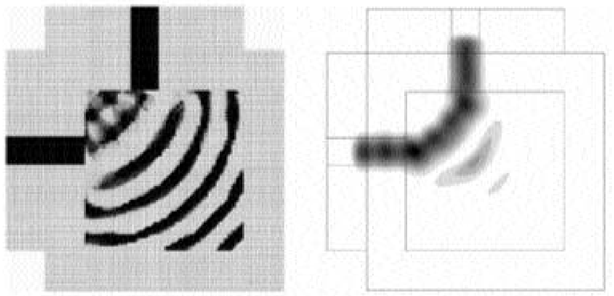


Fig. 6 Configuration of 90^o bend model.



(a) (b)

Fig. 7 Calculated refractive index profile (a) and propagating field intensity (b) for 90^o bend.

Fig. 7(b), we see that most of the input power is transmitted through the 90-degree bend, proving that we can produce an optical waveguide with a sophisticated bend structure by using the developed CAE technique. In the designed bend structure, the wavefront is held constant because the light being propagated outside is accelerated compared with that propagating inside, due to the lower refractive index. Moreover, the outside grating structure suppresses radiating waves. These factors all contribute to the high transmittance of the designed bend structure.

Next, we consider a T-branch waveguide with an arbitrary splitting ratio. **Figure 8** shows the configuration of the T-branch model. The design region size is 4 × 3 $\mu\mathrm{m}^2$, and the other parameters are the same as for the 90-degree bend. For branching with a splitting ratio $\alpha:\beta$, the objective function is specified as follows:

$$\mathit{Minimize}\left|\frac{\beta\,|S_{21}|^2-\alpha\,|S_{31}|^2}{|S_{21}|^2+|S_{31}|^2}\right| \text{ at } \lambda = 1.55\ \mu\mathrm{m} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (6)$$

where S_{n1} is the ratio of the output amplitude at port n to the input amplitude. **Figs. 9** and **10** show the calculated refractive index profile and propagating field intensity for 1:1 and 1:2 branching, respectively. The normalized output powers in the output waveguides are 0.46 and 0.46 for 1:1 branching, and 0.30 and 0.62 for 1:2 branching. Since the ratios of the output power obtained for the optimized structures are close to the splitting ratios given for both branching waveguides beforehand, we can conclude that the developed CAE technique can be successfully applied to the design of T-branch waveguides.

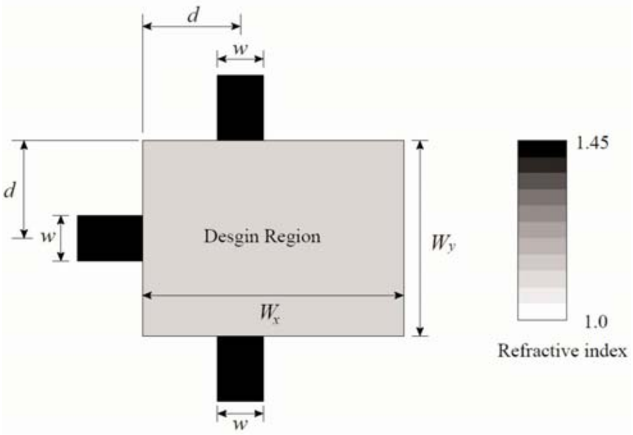
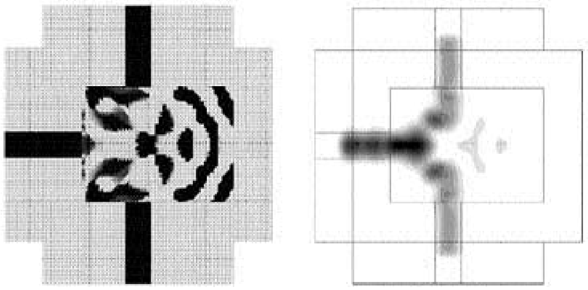


Fig. 8 Configuration of T-branch model.

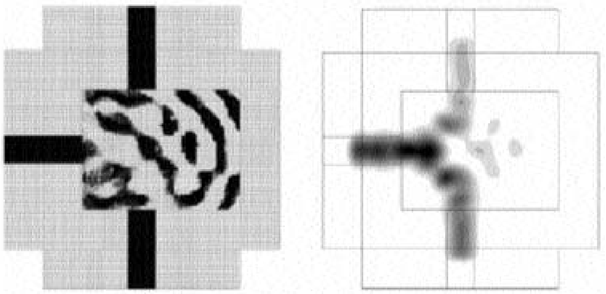
5. Conclusion

We have examined whether a CAE technique based on topology optimization methods can be applied to the design of the internal structures of optical devices. In the verification of dielectric band gap structures, multilayer structures perpendicular to the direction of incident waves, which were expected as the results of calculating the dielectric band gap structure, were obtained for several incident angles. Moreover, since the structures desired for propagating input power efficiently were designed for 90-degree and T-branch waveguides, we were able to confirm the validity of the developed CAE technique.



(a) (b)

Fig. 9 Calculated refractive index profile (a) and propagating field intensity (b) for T-branch waveguide with splitting ratio 1:1.



(a) (b)

Fig. 10 Calculated refractive index profile (a) and propagating field intensity (b) for T-branch waveguide with splitting ratio 1:2.

References

1) Smith, D. R., Pendry, J. B. and Wiltshire, M. C. K. : "Metamaterials and Negative Refractive Index", Science, **305**(2004), 788
2) Pendry, J. B., Holden, A. J., Robbins, D. J. and Stewart, W. J. : "Magnetism from Conductors and Enhanced Nonlinear Phenomena", IEEE Trans. Microwave Theory Tech., **47**(1999), 2075
3) Shelby, R. A., Smith, D. R. and Schultz, S. : "Experimental Verification of a Negative Index of Refraction", Science, **292**(2001), 77
4) Bendsøe, M. P. and Kikuchi, N. : "Generating Optimal Topologies in Structural Design Using a Homogenization Method", Computer Methods in Applied Mechanics and Engineering, **71**(1988), 197
5) Yoo, J., Kikuchi, N. and Volakis, J. L. : "Structural Optimization in Magnetic Devices by the Homogenization Design Method", IEEE Trans. Magnetics, **36**(2000), 574
6) Jensen, J. S. and Sigmund, O. : "Systematic Design of Photonic Crystal Structures Using Topology Optimization: Low-loss Waveguide Bends", Appl. Phys. Lett., **84**(2004), 2022
7) Kiziltas, G., Psychoudakis, D., Volakis, J. L. and Kikuchi, N. : "Topology Design Optimization of Dielectric Substrates for Bandwidth Improvement of a Patch Antenna", IEEE Trans. Antennas Propagation, **51**(2003), 2732
8) Nomura, T., Sato, K., Hirayama, K. and Nishiwaki, S. : "Topology Optimization of Periodic Microstructure of Electromagnetic Material", 6th World Congr. of Struct. and Multidisciplinary Optim. (WCSMO6), No.981(2005), Int. Soc. for Structural and Multidisciplinary Optimization
9) Tsuji, Y., Hirayama, K., Nomura, T., Sato, K. and Nishiwaki, S. : "Design of Optical Circuit Devices Based on Topology Optimization", IEEE Photonics Tech. Lett., **18**(2006), 850

(Report received on Sept. 18, 2006)

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The publisher apologizes for the following omission.
It was corrected as of Jan. 23, 2007.
p.31 <Author introduction : Yasuhide Tsuji and Koichi Hirayama>
<Added> Academic degree : Ph. D.