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Research Report

Effective Calculation Method Based on a Reduction Scheme for Vibration Reduction

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Abstract

In automotive body structural design, Computer Aided Engineering (CAE) has been widely used in order to evaluate noise, vibration, and harshness (NVH). A CAE engineer typically uses a large-scale finite element model exceeding 1 million degrees of freedom to improve the NVH performance criteria. It is, however, difficult for a CAE engineer to propose a good modification candidate for the NVH reduction to an automotive designer, because the FEM calculation is very time-consuming and many design candidates must be considered for a large-scale model. Therefore, quick and effective design calculation procedures are needed to overcome these problems, especially in the recent virtual prototyping development process. In this paper, a new optimal design method using a reduction scheme based on the physical coordinates under many design constraints regarding crash-worthiness is proposed in order to overcome these problems. That is, we determine the appropriate location and additional scalar spring constants by minimizing the acceleration of the observation grid. The effectiveness and availability of this method are confirmed using an example.

Keywords: Structural analysis, Sensitivity analysis, Finite element method, Optimal design method, Reduced model, Frequency response function, Computer aided engieering, Vibration

1. Introduction

New CAE tools based on the concept of First Order Analysis (FOA) usually use beam or panel elements with relatively small numbers of degrees of freedom and which is based on basic theories of structural mechanics. In a wider sense, however, it is considered as an analysis technique that easily provides design engineers with valuable information. Conventional Computer Aided Engineering (CAE) tools, on the other hand, are used by CAE engineers for high-precision, quantitative evaluation by making full use of large-scale Finite Element (FE) models. Conventional CAE provides detailed information by visualizing deformation and stress fields, but is not always capable of providing useful information to design engineers. For example, for noise and vibration problems on the frequency range over 100 Hz, like those found in an automobile, we have to use a large-scale FEM model with more than one million degrees of freedom, in order to consider the effect of local stiffness. The use of more detailed models has contributed to the improvement of the calculation precision but, at the same time, has made it difficult to achieve efficiency in the development phase because of the need for enormous amounts of calculation time. Thus, we need a new analysis technique, that a simple model with a suitable level of precision is constructed and a valuable information is efficiently obtained using this model. We regard this as being part of the technique of FOA because this technique gives design engineers valable information for design based on the calculation results, both easily and promptly.

As a result of our basic studies into FOA, this paper presents an optimal design method that applies the reduction technique using frequency response functions to an FEM model for a vibration problem. In this optimal method, the effectiveness of reinforcement is estimated using added scalar springs as design variables. Chapter 2 first presents the general flow of this optimal design method, then examines the main points of the method in detail, namely, (1) the sensitivity analysis method, (2) the reduction method, and (3) the optimal design method. Chapter 3 demonstrates the validity of the method using a simple example of a body-in-white.

2. Analysis method

2. 1 Flow of optimal design method

We propose an efficient optimization method in which the candidate design variables are selected to create a reduced model, and optimization calculations are carried out on the reduced model. The procedure is as follows:

(1) Set the directions of excitation points and evaluation points, and the target frequency range and its step width.

(2) From an entire large-scale FEM model with n_0 degrees of freedom, select n_1 degrees of freedom and m_1 elements with those degrees of freedom, using inter-node distance and angle in the design domain as thresholds. These elements form what we will call a stiffness sensitivity evaluation model. The elements used in this paper are scalar spring elements added between pairs of nodes.

(3) Using frequency response functions with n_1 degrees of freedom, calculate the stiffness sensitivities of the m_1 elements; then, using those values as a threshold, select m_2 elements as candidate design variables, which consists of n_2 degrees of freedom.

(4) Create a compliance matrix, using frequency response functions of n_2 degrees of freedom.

(5) From the m_2 elements, select m_3 elements as design variables which consist of n_3 degrees of freedom, taking sensitivities and design feasibility into consideration; then create a dynamic stiffness matrix with n_3 degrees of freedom from the compliance matrix obtained in step (4).

(6) Perform optimization calculations while modifying the dynamic stiffness matrix with n_3 degrees of freedom in succession. If the objective is not achieved, return to step (5) and repeat the optimization calculations with new design variables.

(7) Add the obtained stiffness values to the entire model with n_0 degrees of freedom, and perform frequency response calculations to confirm the effectiveness, including the effectiveness on domains outside the target domain.

The number of degrees of freedom analyzed is reduced at each step ($n_0 > n_1 > n_2 > n_3$). The minimum unit of the n_3 degrees of freedom at which the optimization calculation is performed is reduced to four: four-degrees of freedom at excitation point, evaluation point and two points for adding stiffness.

2. 2 Sensitivity analysis method

Assuming the displacement vector to be $\{\boldsymbol{U}\}$, external force vector $\{\boldsymbol{F}\}$, stiffness matrix $[\boldsymbol{K}]$, damping matrix $[\boldsymbol{C}]$, mass matrix $[\boldsymbol{M}]$, and angular frequency ω, an discrete equation of motion of FEM is expressed by

$$([\boldsymbol{K}]+jw\,[\boldsymbol{C}]-w^2[\boldsymbol{M}])\{\boldsymbol{U}\}=\{\boldsymbol{F}\} \quad \cdots\cdots\cdots\cdots\cdots (1)$$

By expressing the matrixes in the parenthesis in Eq. (1) with a dynamic stiffness matrix $[\boldsymbol{B}(\omega)]$ expressed with a complex function for ω, gives us

$$[\boldsymbol{B}(w)]\{\boldsymbol{U}(w)\}=\{\boldsymbol{F}(w)\} \quad \cdots\cdots\cdots\cdots\cdots (2)$$

Let us introduce mutual mean compliance $\boldsymbol{C}$ when we apply an practical periodic external force $\{\boldsymbol{F}\}$ as an evaluation quantity.[2] As the inner product of the vibration response vector $\{\boldsymbol{U}\}$ when $\{\boldsymbol{F}\}$ is applied and the virtual unit period external force $\{\boldsymbol{F}^l\} = \{0, \dots, 0, \boldsymbol{f}_1 = 1, 0, \dots, 0\}^T$ that is applied to the l-th degree of freedom at which to evaluate vibration, $\boldsymbol{C}$ is expressed by

$$C=\{\boldsymbol{F}^l\}^T\{\boldsymbol{U}\}=\{\boldsymbol{U}^l\}^T[\boldsymbol{B}]^T\{\boldsymbol{U}\}=u_l \quad \cdots\cdots\cdots\cdots\cdots (3)$$

where $\{\boldsymbol{U}^l\}$ is the vibration response vector when $\{\boldsymbol{F}^l\}$ is applied. $\{\boldsymbol{F}^l\}$ is the normalized dimensionless quantity that is applied only to the l-th degree of freedom, and $\{\boldsymbol{U}^l\}$ assumes the dimensions of the compliance. Therefore, C is equivalent to the response displacement u_l at the l-th degree of freedom at which we evaluate the vibration in $\{\boldsymbol{U}\}$.

Now, assuming the design variable, that is, the stiffness value of the scalar spring to be added to the degrees of freedom i and j at nodes to be X, the sensitivity of X to u_l is expressed by Eq. (4), $\partial[B]/\partial X$ is 1 for the ii and jj elements, -1 with the ij and ji

elements, and 0 for all other elements, and is finally expressed by Eq. (5)[3].

$$\frac{\partial u_l}{\partial X} = -\{U^l\}^T \frac{\partial [B]}{\partial X}\{U\} \quad \cdots\cdots\cdots\cdots\cdots\cdots (4)$$

$$\frac{\partial u_l}{\partial X} = -(u_i^l - u_j^l)(u_i - u_j) \quad \cdots\cdots\cdots\cdots\cdots (5)$$

Thus, the sensitivity of the stiffness value to the vibration response to be evaluated can be determined analytically with only the vibration response results obtained when two external forces $\{\boldsymbol{F}\}$ and $\{\boldsymbol{F}^l\}$ are applied. This offers the following advantages:

(1) Inverse matrix calculation for the dynamic stiffness matrix is not required. Thus we can obtain the sensitivities using only calculation results of a commercial FE software.

(2) The sensitivity value does not directly depend on the dynamic stiffness matrix and, therefore, is not affected by reduction.

2. 3 Reduction method using frequency response functions

In this section, in order to reduce the dynamic stiffness matrix, we use frequency response functions.[4] One of the reasons for this is that the effect of adding stiffness can be evaluated directly, but another is that the precision of the results for vibration response is assured even after reduction. With mode reduction, the precision of the results depends on the numbers to be reduced and, strictly speaking, the modal results will always contain errors.

The dynamic stiffness matrix $[B]$ in Eq. (2) is separated into the area of the degrees of freedom to be reduced a and the area of the degrees of freedom to be erased b, which are expressed by the following equation[5]:

$$\begin{bmatrix} B_a & B_{ab} \\ B_{ba} & B_b \end{bmatrix} \begin{Bmatrix} U_a \\ U_b \end{Bmatrix} = \begin{Bmatrix} F_a \\ 0 \end{Bmatrix} \quad \cdots\cdots\cdots\cdots\cdots\cdots (6)$$

In Eq. (6), because no external force is applied to the area b, a reduced dynamic stiffness matrix $[\overline{B_a}]$ is expressed by

$$[\overline{B_a}] = [B_a] - [B_{ab}][B_b]^{-1}[B_{ba}] \quad \cdots\cdots\cdots\cdots\cdots (7)$$

We do not, however, use the reduction procedure with Eq. (7) here. The area of the degrees of freedom to be reduced a is very small in comparison with that of the degrees of freedom to be erased b. Therefore the calculation of an inverse matrix $[B_b]^{-1}$ is very time-consuming. Moreover we have to construct a reduced model every time we change the elements to be adopted as design variables. Thus, we first construct the compliance matrix $[H_c]$ of the area of n_2 degrees of freedom selected as candidate design variables. Assuming the area of the degrees of freedom selected as candidate design variables to be c and the area of the other degrees of freedom to be d, $[H_c]$ is expressed by

$$[H_c] = [B_c]^{-1} = \left([B_c] - [B_{cd}][B_d]^{-1}[B_{dc}]\right)^{-1} \quad \cdots (8)$$

$[H_c]$ may be constructed from the entire dynamic stiffness matrix, using the above equation. Alternatively, it can be constructed from the frequency response functions obtained when the unit matrix I_c is applied to the area of the selected degrees of freedom c, as follows:

$$\begin{bmatrix} B_c & B_{cd} \\ B_{dc} & B_{dd} \end{bmatrix} \begin{bmatrix} H_c \\ H_{dc} \end{bmatrix} = \begin{bmatrix} I_c \\ 0 \end{bmatrix} \quad \cdots\cdots\cdots\cdots\cdots\cdots (9)$$

Then, we separate $[H_c]$ into the area of the degrees of freedom to be reduced a and that of the degrees of freedom to be erased e, and determine the reduced dynamic stiffness matrix $[\overline{B_a}]$ through inverse matrix calculation on the element $[H_a]$ in the area a.

$$[\overline{B_a}]\,[H_c]^{-1} \quad \left([H_c] = \begin{bmatrix} H_a & H_{ae} \\ H_{ea} & H_e \end{bmatrix}\right) \quad \cdots\cdots\cdots\cdots (10)$$

Thus, using the reduced dynamic stiffness matrix $[\overline{B_a}]$, Eq. (6) can be expressed as

$$[\overline{B_a}]\{U_a\} = \{F_a\} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (11)$$

In order to maximize the efficiency of the calculation, it would be best to construct $[H_c]$ with Eq. (8). In commercial FEM software, however, the complex values may be divided into real and imaginary parts or Component Mode Synthesis may be used, when calculating the equation of motion (1). Therefore, it is difficult to directly extract the entire dynamic stiffness matrix. In this case, a reduced dynamic stiffness model can be constructed from the vibration response calculation results, using Eqs. (9) and (10). The fact that a reduced model can be calculated using output results only, as the previous section, indicates that this optimal design problem can be solved using not only the results obtained from calculation (FEM), but also ones obtained from experiments.

2. 4 Stiffness optimal design method

We define the optimal design problem in such a way as to minimize the vibration response u_l of the degrees of evaluation point freedom l when the periodic external force $\{\boldsymbol{F}\}$ is applied, using a reduced equation of motion (10), as follows. Here, as a design variable, we use the stiffness value X of the scalar spring to be added, and minimize the real value e resulting from multiplying the complex number u_l by the complex conjugate u_l*, where u_l and e are functions having angular frequency ω as a variable, and for practical applications, vibration reduction within a certain frequency band is required. The reason for this is that reducing the vibration response at a single frequency may simply mean that the resonance frequency has shifted, possibly causing high-level vibration to occur in the vicinity of that frequency. Thus, we solve a single optimization problem such as that shown below, with E as an objective function, which results from adding the values of $e_m = e(\omega_m)$ made discrete into M elements with a certain frequency range and step, i.e., the products of $u_{lm} = u_l(\omega_m)$ and $u_{ml}^* = u_l^*(\omega_m)$ at individual frequencies. We set the upper limit $\overline{X}$ and the lower limit $\underline{X}$ of the design variable as side constraints.

$$\min E(X) = \sum_{m=1}^{M} e_m(X) = \sum_{m=1}^{M} u_{lm}(X) \cdot u_{lm}^*(X) \quad \cdots\cdots (12)$$

$$\text{subj. to} \quad \overline{X} \geq X \geq \underline{X}$$

The sensitivity S of the design variable X to the objective function E is expressed by

$$S = \frac{\partial E}{\partial X} = \sum_{m=1}^{M} \frac{\partial e_m}{\partial X} = \sum_{m=1}^{M} u_{lm} \frac{\partial u_{lm}^*}{\partial X} + \frac{\partial u_{lm}}{\partial X} u_{lm}^*$$

$$= \sum_{m=1}^{M} u_{lm} \left(\frac{\partial u_{lm}}{\partial X}\right)^* + \frac{\partial u_{lm}}{\partial X} u_{lm}^* = \sum_{m=1}^{M} 2\mathrm{real}\left(u_{lm}\frac{\partial e_{lm}}{\partial X}\right) \quad \cdots\cdots (13)$$

Using Eq. (5), Eq. (13) is expressed by

$$S = \sum_{m=1}^{M} -2\mathrm{real}\,\{u_{lm}(u_{im}^l - u_{jm}^l)(u_{im} - u_{jm})\} \quad \cdots\cdots (14)$$

where u_{im} and u_{jm} are the response displacements at the degrees of freedom i and j to which the stiffness X is added when $\{\boldsymbol{F}(\omega_m)\}$ is applied, and u^l_{im} and u^l_{jm} are the response displacements (compliances) at the degrees of freedom i and j when the normalized unit periodic external force $\{\boldsymbol{F}^l(\omega_m)\}$ that is applied only to the l-th degree of freedom is applied.

By calculating the sensitivity with Eq. (14) while reducing the dynamic stiffness matrix $[\overline{B_a}]$ with Eq. (10) sequentially with respect to X, we can solve the unconstrained optimization problem of Eq. (12) with a Quasi-Newton method.[6] If there is more than one design variable X, the sensitivity S is expressed by a vector, and $[\overline{B_a}]$ must be reduced as a matrix containing all the degrees of freedom of the elements constituting the stiffness X to be added.

3. Numerical calculation examples

We will now demonstrate the validity of this optimal design method by using an FE model consisting of the shell elements shown in **Fig. 1**. Here, we proceed with a study that follows the flow presented in **2. 1**, using a frequency response calculation by the commercial FEM software (MSC/NASTRAN). The problem that we solve involves minimizing the acceleration $\ddot{u}_l$ in the x direction in the glass portion (marked with a circle (○) in the figure) when a unit periodic external force F is applied to the region in the direction shown in Fig. 1. Here, we convert the equations given above, in which displacements are used as evaluation quantities, into formulas in which accelerations are used as evaluation quantities, using the relation $\ddot{u}(t) = \partial^2 u(t)/ut^2 = -\omega^2 u(t)$. Because the response acceleration peaks from 140 through 150 Hz, the frequency range for optimal design is set from 130 through 155 Hz, and the step width is set to 1 Hz.

We assume that the area excluding the glass

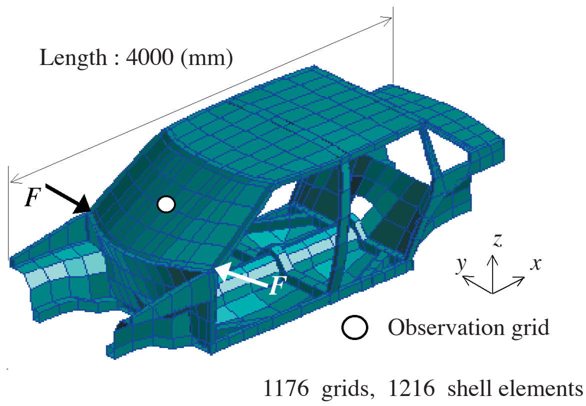

Fig. 1 FEM model of a body-in-white.

portion is the object of the design, and extract the nodes at which to study the stiffness sensitivity, as well as the elements connecting these nodes (stiffness sensitivity evaluation model). The model shown in Fig. 1 is a simple one in which the inter-node gaps are relatively large, so that we set the nodes and elements constituting the individual sides of the shell elements as defaults, and set the other thresholds as (1) elements whose inter-node length L is from 250 to 300 mm and (2) elements connecting nodes whose angle θ from the individual sides of the shell elements in Fig. 1 is from 30° to 180°. The resultant stiffness sensitivity evaluation model is shown in **Fig. 2**. This model has more than 3000 elements. As shown in Fig. 2, the inter-node stiffness in a portion in which there are no shell elements can be evaluated. This is useful when considering the addition of a structure to a region in which there is currently no structure, rather than modifying an existing structure. The stiffnesses that we consider here are those in the three translation directions, and the number of stiffness values with which to calculate the sensitivity is equal to the number of elements × 3 (number of directions), that is, approximately 10000.

From the frequency response calculation results obtained by FEM, we can calculate the sensitivity of the stiffness value X of each element in the stiffness sensitivity evaluation model shown in Fig. 2, using Eq. (9). Here, from the sensitivity calculation results, we select the elements as candidate design variables, using a value equal to 10% of the maximum as a threshold. The selection results are shown in **Fig. 3**. In Fig. 3, the portions indicated by the heavy lines are those elements that have high sensitivity. As a result, the regions regarded as candidate design variables are reduced from 1176 nodes to 107 nodes (321 degrees of freedom) and from 3577 elements to 75 elements (225 candidate design variables).

For the degrees of freedom at nodes, at excitation points, and at the evaluation points of the elements selected as described above, we construct an inertance matrix. This can be determined by using an FEM frequency response calculation when a unit input is applied at each degree of freedom, as shown in Eq. (9). From this matrix, we extract the elements with the degrees of freedom, the degrees of freedom at excitation points, and those at evaluation points as design variables, and then perform an inverse matrix calculation with Eq. (10) to construct a reduced dynamic stiffness matrix.

Using the reduced model, we solve the optimization problem for the single design variable given in Eq. (12). In this example, many of the elements selected as candidate design variables as shown in Fig. 3 are symmetrical, so we think it appropriate to handle these elements as identical design variables. If we consider this, the reduced model will be very small with a total of 7 degrees of freedom; 4 degrees of freedom constituting the

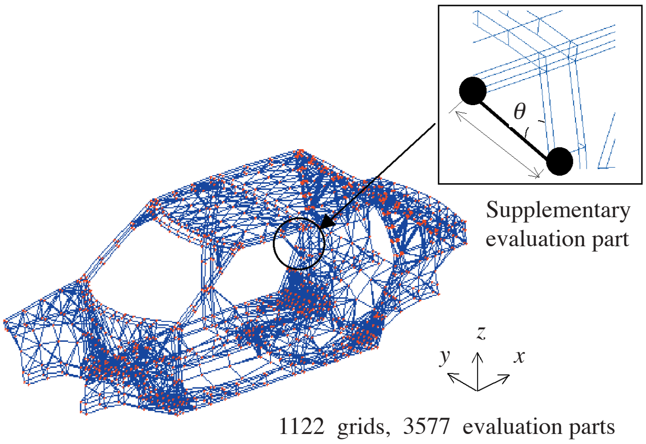

Fig. 2 Modified stiffness sensitivity evaluation Model.

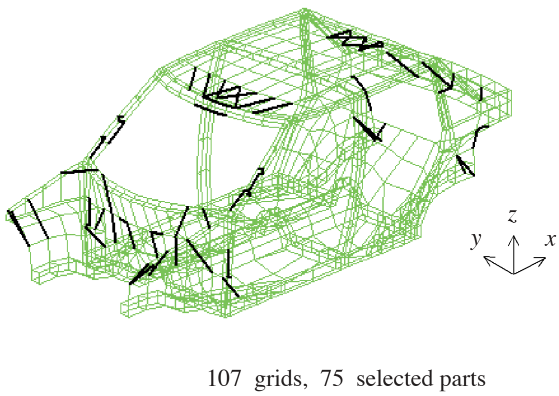

Fig. 3 High-sensitive parts.

design variable, 2 degrees of freedom at the excitation points, and 1 degree of freedom at an evaluation point.

The results of extracting those regions in which the response acceleration $\ddot{u}_l$ of the degree of freedom at an evaluation point can be reduced by 25% or more are shown in **Fig. 4**. The regions indicated by the heavy lines are those in which the vibration response of the glass surface can be reduced by adding the calculated individual optimal stiffness values in the prescribed direction. For example, in region *a* shown in Fig. 4, an optimal stiffness value of 1100 N/mm in the *z* direction is calculated. The results of adding a scalar spring with this value to the portion that is symmetrical to *a*, and then calculating the frequency response, are shown in **Fig. 5**. These results show that the vibration response on the glass surface is reduced in the specified frequency band. Similar vibration reduction effects can be confirmed in the other regions shown in Fig. 4.

Based on the results obtained from the stiffness optimization calculation shown in Fig. 4, we confirmed the vibration reduction effect when a reinforcement having a specific structure is added. As a reinforcement, we considered a steel plate with regions a and b in Fig. 4 as its two sides and with a thickness of 3 mm, and added it as a shell element to the FEM model shown in Fig. 1. The results of calculating the vibration response on the glass surface are shown in **Fig. 6**. From these results, we can confirm that a reduction effect similar to that shown in Fig. 5 can be achieved in the specified frequency band. The weight of the reinforcement is approximately 1 kg, which increases the total weight of 122 kg by less than 1%.

4. Conclusions

In this paper, we proposed a new optimal design method that provided effective information regarding reinforcement of an automotive body structure in the final design phase, and confirmed the availability of the method using a practical example. The results obtained here are summarized as follows:

(1) The dynamic mutual mean compliance was formulated as the measure of the target vibration,

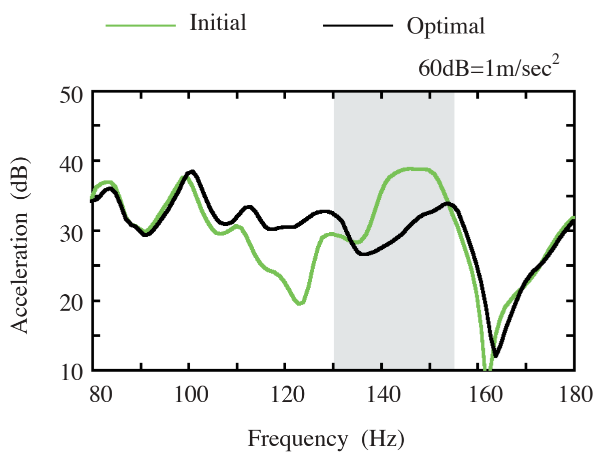

Fig. 5 Effect of reduction by the additional stiffness.

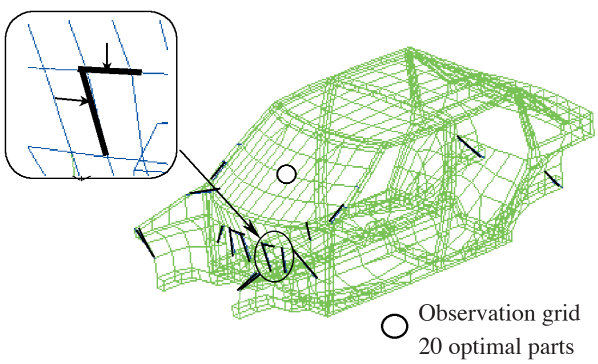

Fig. 4 Optimal parts.

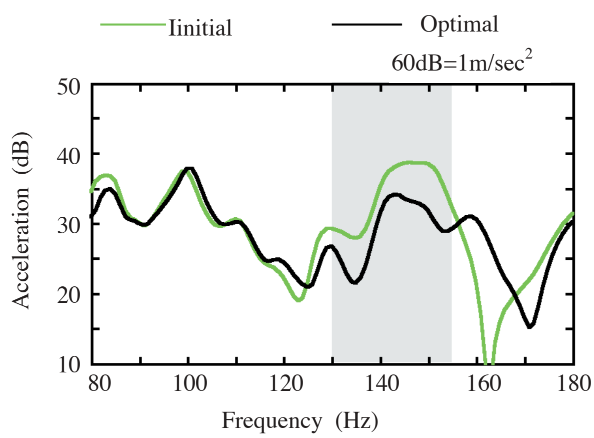

Fig. 6 Effect of reduction by the additional structure.

and the selection method of practical evaluation parts under the measure of sensitivities of the target vibration was proposed.

(2) The new reduced method based on frequency response functions was presented.

(3) The optimization method, where the square of acceleration and additional scalar spring constants were taken an objective function and design variables, respectively, was proposed.

(4) It was confirmed that some appropriate characteristic information such as the local locations, the directions, and the additional scalar spring constants was obtained within a short time in an example.

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