

First Order Analysis for Automotive Suspension Design

Research
Report

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Abstract

The elastic properties of a torsion beam suspension determine the kinematic performance of the suspension. In this paper, we introduce a design program for the torsion beam suspension based on the FOA concept, and confirm its usefulness using simple examples and experimental evaluations.

Keywords Suspension system, Torsion beam, Design, FOA

1. Introduction

Torsion beam suspension, which is often used as the rear suspension in FF-type vehicles, offers the advantages of having a small parts count and a simple configuration (**Fig. 1**). This suspension consists of two trailing arms that are mounted to the body with bushings and a single torsion beam that connects these arms together. The torsion beam must be stiff enough to support lateral forces during cornering and, at the same time, be flexible enough to allow the right- and left-hand wheels to displace differently when driving over a bump. When a

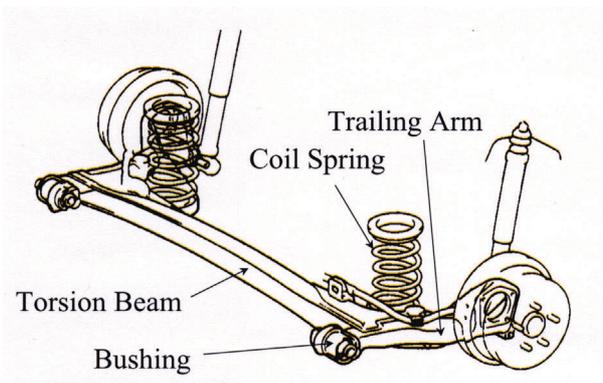


Fig. 1 Torsion beam suspension.

torsion beam suspension is being designed, therefore, we must consider the relative balance between the stiffness and the flexibility of the constituent members, in spite of the simple configuration. In the suspension planning stage, it is necessary to apply performance prediction calculation for alignment changes and compliance steer, with this being even more important in the case of a torsion beam suspension, which is influenced by the stiffness of each of its constituent members.

To calculate the mechanical properties of a torsion beam suspension, finite element analysis and mechanical analysis¹⁾ have been used to date. For many design engineers, however, the planning-stage construction of a finite element model or a complicated mechanical analysis model that incorporates the elastic properties of its constituent members, and then performing calculations using that model, requires a tremendous amount of time and technical skill and is often difficult to accomplish.

Thus, this research proposes First Order Analysis²⁾ as a CAE tool for design engineers that can be used with no special knowledge or skills in modeling or analysis. We also describe a design tool that was developed to calculate the properties of a torsion

beam suspension, and confirm the effectiveness of the tool using examples and experimental verifications.

2. FOA for torsion beam suspension

2.1 Hierarchical data structure of the object of design

In conventional types of CAE, the configuration of a model is such that the positional information of its members and the information for the members themselves (including cross-sectional and material properties) are combined. In FOA, the object of the design is configured with a hierarchical data structure like that shown in **Fig. 2**, thereby achieving both efficient and simple operability. More specifically, the uppermost layer of the hierarchy handles only information related to the entire configuration, including the layout. If the design engineer wishes to study one of the constituent members in more detail, he or she can move to a lower layer to study that specific member only. Finally, the design engineer can manipulate the object of design specialized in a specific member to incorporate that object into the top layer, i.e., the information for the entire suspension. In this way, the design engineer can analyze the effect of each specific member on the entire suspension.

Such a hierarchical data structure can easily be

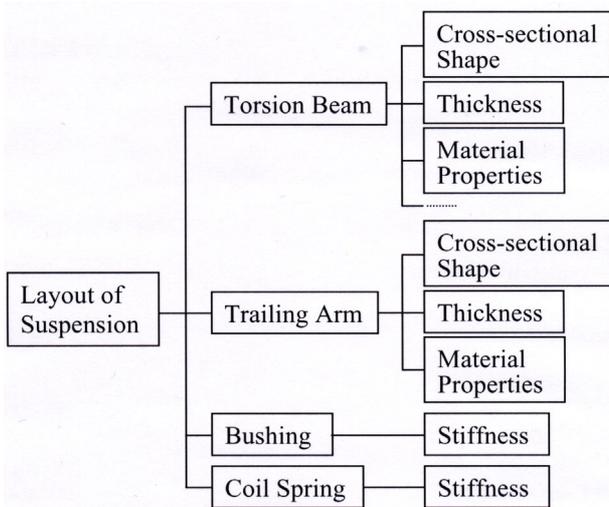


Fig. 2 Hierarchical data structure of torsion beam suspension.

achieved by using spreadsheets that are created on a personal computer using the Microsoft Excel spreadsheet software with which the design engineer will already be familiar.

2.2 Configuration of the design tool

An example FOA configuration is shown in **Fig. 3**. Figure 3 (a) shows the sheet for the top layer. Simply by moving the appropriate scroll bars on the sheet, the designer can enter a design plan for the entire configuration, such as the layout of each member. If, for example, the designer needs to consider the torsion beam itself, he or she merely has to click the displayed torsion beam to replace the displayed sheet with that shown in Fig. 3 (b). Using this sheet to enter information specific to the torsion beam, including the cross-sectional shapes, material properties, amounts of curvature, and whether to add reinforcements, it is possible to study the entire shape of the torsion beam and to incorporate that information into the design plan for the entire suspension while considering the dynamic properties specific to open cross-sectional shapes, using degree-of-freedom reduction calculation, described later. If a cross-sectional shape is to be changed here, the cross-sectional shape on the sheet may be clicked to display the edit sheet specifically for open cross-sections, shown in Fig. 3 (c). This sheet not only allows the designer to perform cross-sectional shape editing with the mouse, in much the same way as with general CAD programs, but also gives him or her access to the cross-section property calculation functions, described later, such as second moment of inertia, polar moment of inertia, centroid, and shear center. This makes it possible to study a cross-sectional shape while referencing the property values displayed after each design change. For another component, the trailing arm, the same procedure can be used to study its design. For the bushings and the coil springs, their positions can be determined in the top layer, after which only the bushing stiffness and the spring constant need to be determined.

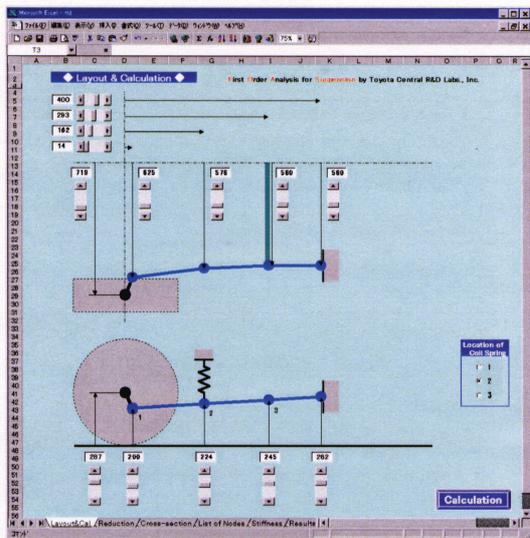
2.3 Characteristic analysis of objects of design

After a design has been set using the procedure described above, returning to the top layer of this design tool and then clicking the calculate button causes the suspension properties such as compliance

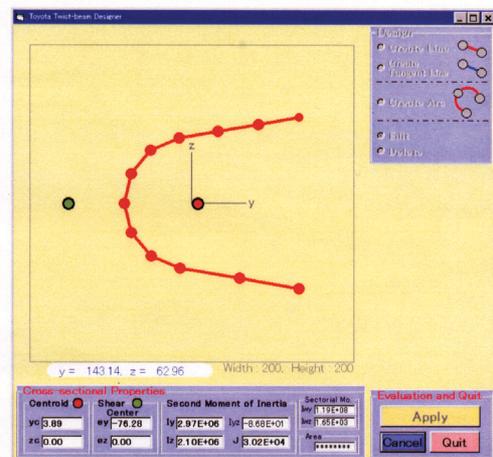
and alignment changes to be calculated and displayed on the sheet shown in Fig. 3 (d). Here, after a suspension is represented by multiple beam elements and stiffness matrixes and appropriate boundary conditions are supplied, the compliance properties are calculated using linear finite element analysis after which alignment changes are calculated using non-linear finite element analysis. In both calculations, a Toyota-CRDL developed

solver that runs on a personal computer is used. The amount of time needed for calculation is at most tens of seconds.

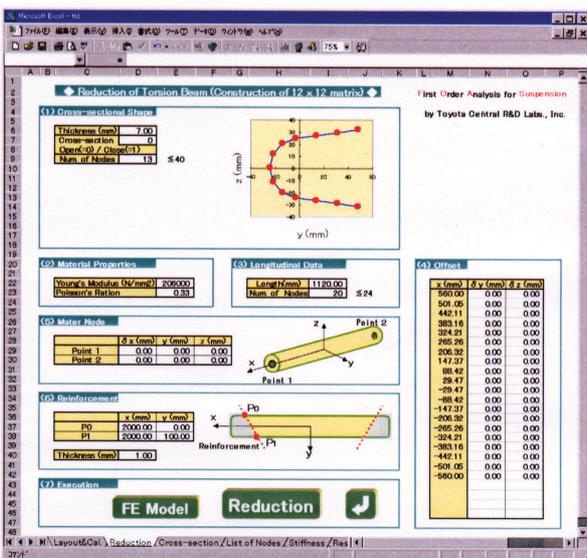
The above has explained the procedure for using this design tool based on the concept of FOA. For torsion beams with open cross-sectional shapes, consideration has been given to assisting design engineers by providing dedicated calculation functions, as mentioned earlier. The calculation



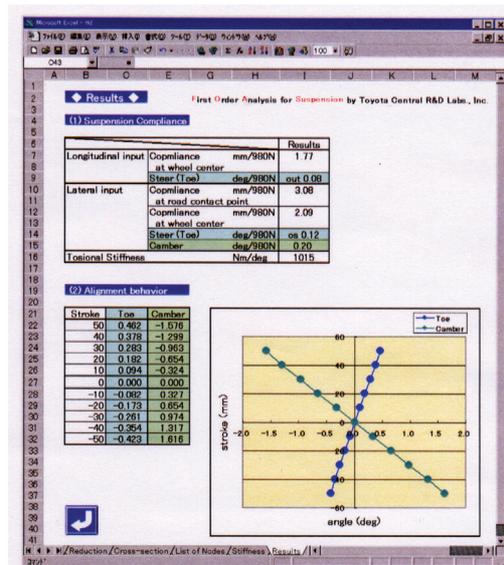
(a) Top sheet



(c) Sheet of CAD for cross-sectional design



(b) Sheet for torsion beam design



(d) Sheet for numerical results

Fig. 3 Design program for torsion beam suspension.

method is explained briefly below.

2.4 Cross-sectional property calculation

In general, torsion beams have an open cross-sectional shape to reduce their torsional stiffness. Among the properties of an open cross-section, the second moment of inertia and the polar moment of inertia, which indicate the stiffness balance of the torsion beam, are important indexes. Also important is the shear center that has a great effect on alignment changes. The following describes the calculations that can be performed using this program.

As shown in Fig. 4, a cross-section of a torsion beam is represented by a group of n_{max} line elements on the y - z coordinate system. The thickness of the n -th line element is assumed to be t_n , each line element is subdivided into k_{max} elements, the centroid of each of the resulting small-line elements is assumed to be (y_{ck}, z_{ck}) , and the length is assumed to be Δs_k . According to Oden³⁾, the second moment of inertia in this case is given by

$$I_y = \sum_{n=1}^{n_{max}} \sum_{k=1}^{k_{max}} \frac{z_{ck}^2}{1 - y_{ck}/R} t_n \Delta s_k \dots \dots \dots (1)$$

$$I_z = \sum_{n=1}^{n_{max}} \sum_{k=1}^{k_{max}} \frac{y_{ck}^2}{1 - y_{ck}/R} t_n \Delta s_k \dots \dots \dots (2)$$

$$I_{yz} = \sum_{n=1}^{n_{max}} \sum_{k=1}^{k_{max}} \frac{y_{ck} z_{ck}}{1 - y_{ck}/R} t_n \Delta s_k \dots \dots \dots (3)$$

where R denotes the radius of curvature of the torsion beam in the longitudinal direction. On the

other hand, the polar moment of inertia is given by

$$J = \frac{1}{3} \sum_{n=1}^{n_{max}} \sum_{k=1}^{k_{max}} \Delta s_k t_n^3 \dots \dots \dots (4)$$

for an open section, and by

$$J = \left(\sum_{n=1}^{n_{max}} \sum_{k=1}^{k_{max}} r_k \times \Delta s_k \cdot i \right)^2 \sum_{n=1}^{n_{max}} \sum_{k=1}^{k_{max}} \frac{\Delta s_k}{t_n} \dots \dots \dots (5)$$

for a closed section

where r_k denotes the direction vector of the small-line element (s_k^n, s_{k+1}^n) , Δs_k the vector connecting the ends of the small-line element (s_k^n, s_{k+1}^n) , and i the unit vector in the beam longitudinal direction.

Assuming that the centroid is at the origin, the position of the shear center is

$$e_y = -2 \frac{I_y I_{oz} - I_{yz} I_{oy}}{I_y I_z - I_{yz}^2}, e_z = 2 \frac{I_y I_{oy} - I_{yz} I_{oz}}{I_y I_z - I_{yz}^2} \dots \dots \dots (6)$$

where

$$I_{oy} = \sum_{n=1}^{n_{max}} \sum_{k=1}^{k_{max}} Q_y r_k \times \Delta s_k \cdot i$$

$$= \sum_{n=1}^{n_{max}} \sum_{k=1}^{k_{max}} \sum_{n'=1}^n \sum_{k'=1}^k \frac{z_{ck'}}{1 - y_{ck'}/R} t_n \Delta s_k r_{k'} \times \Delta s_{k'} \cdot i$$

\dots \dots \dots (7)

$$I_{oz} = \sum_{n=1}^{n_{max}} \sum_{k=1}^{k_{max}} Q_z r_k \times \Delta s_k \cdot i$$

$$= \sum_{n=1}^{n_{max}} \sum_{k=1}^{k_{max}} \sum_{n'=1}^n \sum_{k'=1}^k \frac{y_{ck'}}{1 - y_{ck'}/R} t_n \Delta s_k r_{k'} \times \Delta s_{k'} \cdot i$$

\dots \dots \dots (8)

Q_y and Q_z are the static moments of inertia.

2.5 Finite element models and degree-of-reduction calculation

To incorporate a torsion beam design plan into a design plan for an entire suspension, we use degree-of-freedom reduction calculation. First, we create a three-dimensional representation of the torsion beam by longitudinally enlarging the cross-sectional shape drawn using the editing program mentioned earlier. We displace this shape with a finite element model within the program, and then automatically convert the stiffness matrix into a stiffness matrix with 12 degrees of freedom at both ends of the torsion beam, using degree-of-freedom reduction calculation. The shape of the finite element model thus created can be confirmed on a three-dimensional display like that shown in Fig. 5. In this model, the shell elements constituting the torsion beam are connected to two degenerate points, using sufficiently stiff beam elements, to enable the beam to be incorporated stiffly with the trailing arms and also enable reliable

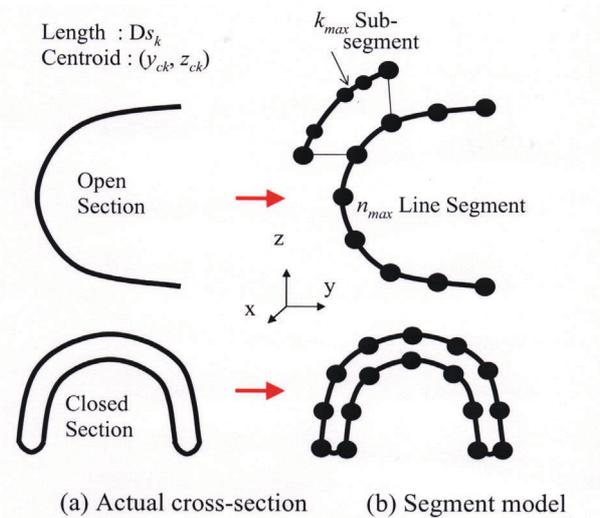


Fig. 4 Cross-sectional model using line segments.

load transmission.

We use a finite element model for the torsion beam in order to incorporate the degrees of design freedom specific to this suspension type, such as (1) the cross-sectional property that states that the centroid is displaced from the shear center, (2) the amount of curvature in the torsion beam, and (3) the addition of reinforcements. In addition, by reducing the degrees of freedom of the finite element model and incorporating them, consideration is given to reducing the calculation time and improving the practicability.

To reduce a finite element model into a stiffness matrix with 12 degrees of freedom, we used the approach proposed by Guyan.⁴⁾ Let the stiffness matrix for a finite element model be K , then, using the symbol m to indicate a node that will remain as a reduction point, and the symbol s , which indicates the node to be removed, K can be represented by the following submatrix:

$$K = \begin{bmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{bmatrix} \dots\dots\dots (9)$$

Using the following transformation matrix,

$$T = \begin{bmatrix} I_{mm} \\ -K_{ss}^{-1}K_{sm} \end{bmatrix} \dots\dots\dots (10)$$

the stiffness matrix in which the degrees of freedom related to the reduction points will be expressed by $K^R = T^T K T \dots\dots\dots (11)$

Finally, this matrix is automatically incorporated between the trailing arms.

3. Verification

3.1 Operability

Table 1 shows the results of comparing this design tool with conventional CAE from the viewpoint of operability. When using this design tool, no more than two to three hours is needed for the calculation. For a parameter study associated with a simple design change, an answer can be obtained in just a few minutes. Design engineers can, therefore, use the tool for their daily design study.

The fact that this tool can be run on a personal computer using familiar spreadsheet software means that there is no need for any special knowledge or skills in handling general-purpose CAE software on an engineering work station (EWS). Similarly, there is no need to bring in other departments. We believe that this offers the possibility of allowing "just a little bit of inspiration" to be examined mechanically and that the tool can be used midway through a design review. The individual sheets of this design tool can be easily disconnected and connected, so that they could easily be applied to other suspension

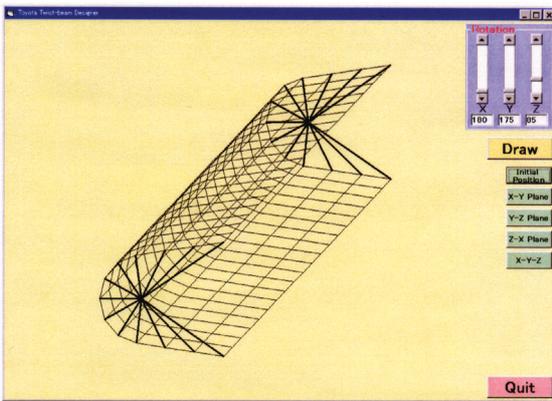


Fig. 5 Finite element model of torsion beam on 3D confirmation program.

Table 1 Comparison between FOA and current CAE.

| | FOA | Present CAE |
|--------------|-------------------|--------------------------|
| Design phase | Concept design | Detail design |
| User | Designer | Analyst |
| Hardware | Personal Computer | Engineering Work Station |
| Time | 2~3 hours | 2~3 weeks |

Table 2 Properties of suspension compliance.

| | | Numerical results | Experimental results |
|--------------------|--|-------------------|----------------------|
| Longitudinal input | Compliance mm/980N at wheel center | 1.78 | 1.68 |
| | Toe deg/980N | out 0.08 | out 0.08 |
| Lateral input | Compliance mm/980N at road contact point | 3.08 | 3.46 |
| | Compliance mm/980N at wheel center | 2.09 | 2.03 |
| | Toe deg/980N | os 0.12 | os 0.11 |
| | Camber deg/980N | 0.20 | 0.26 |

types.

3. 2 Calculation precision

We will verify the calculation precision for the compliance properties and alignment changes calculated with this design tool. **Table 2** lists the numerical and experimental results obtained for various compliance properties, while **Fig. 6** shows the numerical and experimental results for alignment changes if the right- and left-hand wheels are displaced in reverse phases.

From Table 2 and Fig. 6, we can see that the numerical results and experimental results match well and that the design tool can be used satisfactorily in the planning stage, in which the acceptability of the design plans are decided.

4. Conclusions

In this report, we described a design tool that was developed for torsion beam suspensions, and which is an application of First Order Analysis to suspension mechanical design. We then verified the effectiveness of the tool using examples. We can draw the following conclusions:

(1) This design tool enables design engineers to quickly perform basic design and property evaluation in the suspension planning stage, simply and easily using a personal computer.

(2) Experiments have shown that the calculated suspension properties are of a degree of precision sufficient to allow us to judge the acceptability of design plans in the planning stage.

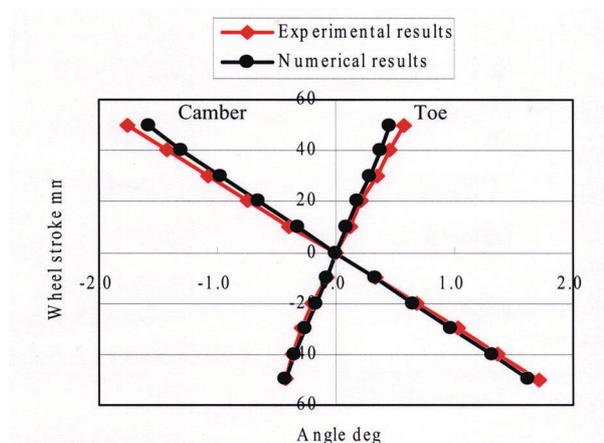


Fig. 6 Alignment behavior with respect to rolling motion.

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