Special Issue First Order Analysis

Research Report

Sizing Optimization Using Response Surface Method in FOA

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Abstract

Computer Aided Engineering (CAE) has played an important role in automotive development. CAE numerically estimates the performance of automobiles and proposes alternative ideas that lead to higher performance without building physical prototypes. However, current CAE can not usually be used in the initial design phase due to its sophisticated, difficult, and complex functions and characteristics. First Order Analysis (FOA) has been proposed to provide a new type of CAE for design engineers. In this report, we present a sizing optimization method based on the response surface method. This method can comparatively deal with any objective functions such as the structural stiffness and the weight of a structure. First, we briefly review the outlines of the response surface method, and benefits when using this method in the numerical analysis. Next, a sizing optimization program is introduced, and its functions are explained in detail using a simple two-dimensional optimization problem. Finally, some examples are provided to confirm the availability of the method proposed here.

Keywords: Computer aided engineering, Optimal design, Sizing optimization, Design of experiment, Response surface methodology

1. Introduction

In the field of automotive development, advances in Computer Aided Engineering (CAE) nowadays allow us to quantitatively estimate the performance of automobiles to some degree, and to make structural proposals to improve the performance. As a result, CAE greatly contributes to reducing the number of prototyping products and the development period. Current CAE are, however, used to perform numerical experiments that replace prototyping with analyses. For this reason, it is required CAE to offer a high level of quantitative precision in the current. Since modeling and analysis methods have become complicated, accordingly non-specialist analysts can hardly execute these methods. Moreover, calculation in the current CAE requires an enormous amount of time for both the modeling and analysis. For the reasons described above, we often come across situations where it is difficult for design engineers themselves to use CAE at the concept design stage, which is the first stage of the design process.

In response to such situations, First Order Analysis (FOA)[1] has been proposed as a new CAE concept to enable design engineers themselves to easily use CAE in the initial design stage.

Using FOA tools, design engineers can create good design candidates in the concept design stage, while analyzing physical properties of them simultaneously. And, in addition to this, to assist these design engineers, FOA offers optimal calculation techniques that encourage the use of topology optimization and sizing optimization.[2]

In FOA, we use an optimization technique based on a response surface methodology that offers speed and convenience as the size optimization.

In this paper, we briefly explain the response surface methodology applied to sizing optimization in FOA, and confirm its validity and effectiveness using simple numerical examples.

2. Optimization using the response surface methodology

Response surface methodology, which has recently attracted attention as an optimization technique in the field of numerical analysis, was advocated by Box & Wilson in 1951.[3] Later, due to the research of Taguchi method[4] by Taguchi and various design of experiments by Myers & Montgomery, et al., it has developed into the response surface methodology[5] that we know today. This technique has been put to practical use in the field of quality engineering for purposes such as product process optimization and variation reduction, especially in the United States.

In around 1995, response surface methodology was first applied to the field of numerical analysis. In the United States, it was used by Haftka, et al., to optimize[6] composite wings for use in ultra-high capacity airplanes. At the same time, in Japan, Shiratori, et al., applied it to the optimization[7] of automotive seat frames, but with a totally different concept.

2. 1 Response surface methodology in numerical analysis

The response surface methodology is a type of optimization that applies an approximation technique to the objective and other functions of an optimization problem. For approximation, it uses a function called a response surface. A response surface is a function that approximates a problem with design variables and state quantities, using several analysis or experiment results. In general, the design of experiments is used for analysis or experiment point parameter setting, and the least square method is used for function approximation.

Figure 1 shows the flow of conventional optimization. In the field of numerical analysis, this technique is commonly used to determine the sensitivities for individual numerical models and to perform optimization using the sensitivities and numerical models. This technique attains convergence by repeating numerical analysis and sensitivity analysis until the optimal solution is obtained. As a result, if the model scale is large, an immensely long calculation time and huge amount of computing resources are required, making it impractical to even attempt the optimization. For problems with very high non-linearity and for multimodal problems, there may be cases in which no solution can be found because of problems such as the inability to obtain sensitivities or a lapse into a local solution. To solve such problems with conventional optimization, the response surface methodology has been adopted.

With the response surface methodology, optimization conditions are first set, and then a response surface is created between the design variables and objective functions or constraint conditions. And, by using the response surface, an optimal solution can be found with a conventional optimization technique. Because only a very simple function, called a response surface, is used in the optimization calculation, it can be completed very quickly (**Fig. 2**).

It should be noted that the precision of an optimal solution depends on the approximation precision of the response surface.

As described earlier, with the response surface

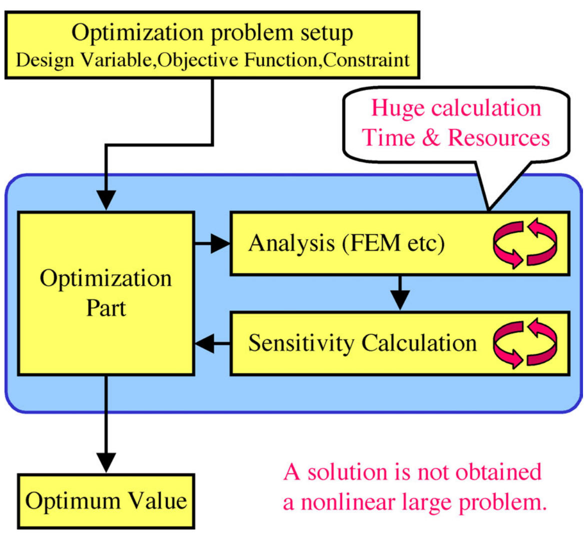

Fig. 1 Conventional optimization.

methodology, parameter setting is generally performed with design of experiments, and the numerical analysis is repeated as many times as set. Then, based on these results, the objective functions or constraint conditions are approximated with a response surface. Thus, by using design of experiments the reliability of the response surface can be increased.

2. 2 Response surface

The response surface is an approximation of the relational expression of the response y predicted from n ($n > 1$) design variables x_i (where $i = 1, ..., n$) (Eq. (1)).

$$y = f(x_1 \cdots x_n) + \varepsilon \quad \cdots\cdots (1)$$

In general, for this function f, a polynomial is often used because it is easy to handle; a non-linear function that can be linearized through variable transformation (such as an exponential function) may also be used.

In special cases, neural networks, splines, and Lagrange's interpolations can also be called response surfaces. These functions are, however, disadvantageous in that they do not correspond to the effective design of experiments and that they cannot be evaluated statistically.

2. 2. 1 Least square method

For response surfaces, linear functions (and functions that can be linearized) are advantageous in that their coefficients can be determined easily using the least square method and that statistical evaluation can be conducted on them once their coefficients have been determined. For this reason, function approximations using the least square method are used most often with the response surface methodology.

If a quadratic polynomial is used as a response function, the response surface is given by Eq. (2):

$$y = \beta_0 + \sum_{i=0}^{n} \beta_i x_i + \sum_{i=0}^{n} \beta_{ii} x_i^2 + \sum_{i<j}^{n} \beta_{ij} x_i x_j \quad \cdots\cdots (2)$$

Representing the above with two variables for simplicity gives Eq. (3):

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_2^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2 \quad \cdots\cdots (3)$$

If, in this expression, we replace the second degree terms with a single variable ($x_1^2 = x_3$, $x_2^2 = x_4$, $x_1 x_2 = x_5$), respectively, this expression is converted into a multi-variable, linear expression. Such conversion is applicable to a third- or higher-degree polynomial.

If linearization is performed in this way, a linear regression model can be represented by Eq. (4), assuming that the number of experiment points (analysis points) is n and the number of design variables is k:

$$y = X\beta + \varepsilon \quad \cdots\cdots (4)$$

$$y = \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{Bmatrix} X = \begin{Bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{Bmatrix} \beta = \begin{Bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{Bmatrix} \varepsilon = \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{Bmatrix}$$

By minimizing the sum of squares of this error, the unbiased estimator b of the coefficient β is given by Eq. (5):

$$b = (X^T X)^{-1} X^T y \quad \cdots\cdots (5)$$

In general, the coefficient of determination R^2 is used to decide whether a regression model is appropriate. The coefficient of determination R^2 provides an exact match if it is 1 and, if the residual increases, R^2 decreases in the range from 1 to 0. As the number of variables increases, the residual decreases, so that the coefficient of determination increases in value. For this reason, to obtain a more precise regression model judgment, the coefficient of determination adjusted for the degrees of freedom R_{ad}^2 (Eq. (6)) is used, which is used for comparing the residual per unit degree of freedom.

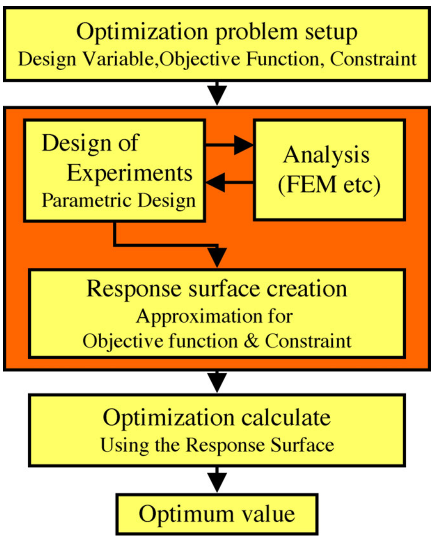

Fig. 2 Response Surface Methodology (RSM).

$$R^2_{ad} = 1 - \frac{SSE / (n\text{-}k\text{-}1)}{Syy / (n\text{-}1)} \quad \cdots\cdots\cdots\cdots\cdots\cdots (6)$$

Residual sum of squares : $SSE = y^T y - b^T X^T y$

Response y fluctuation : $Syy = y^T y - T^2/n$

The validity of individual variables to a response can be determined by conducting t- and F-tests.

2. 3 Design of experiments

Design of experiments is a technique for setting an efficient experiment point parameter. This is equivalent to establishing a parameter for creating a better regression expression. This regression expression itself will be a response surface using the least-square method, as explained in the preceding section.

In design of experiments, all variables, even continuous ones, are thought of as being discrete "levels". By discretizing variables in this way, a design of experiments is advantageous in that it can reduce the number of combinations and is resistant to noise.

As a design of experiments, many techniques have been proposed such as full factorial design, orthogonal design, central composite design (CCD), and the D optimization criterion. In general, orthogonal design is often used for a linear polynomial and a CCD is often used for a quadratic polynomial. A method that uses orthogonal design is, however, thought to be easier to use because of the ease with which levels can be allocated and because of its efficiency. An orthogonal design is outlined here.

2. 3. 1 Orthogonal design

In an orthogonal design, parameter setting involves allocating levels by using an orthogonal array. **Table 1** describes an L8 orthogonal array with two levels. In this array, the rows indicate the number of experiments. Parameters can be set easily by allocating variable levels to the individual columns as instructed.

For a linear function, a two-level orthogonal array is used; for a quadratic function, a three-level orthogonal array is used. An orthogonal array for a function of a third or higher degree can be created. For a multi-level orthogonal array, however, experiments will not be orthogonal and, therefore, for a high polynomial of a second or higher degree, an orthogonal polynomial is required. In general, Chebyshev's orthogonal polynomial is often used. By using both an orthogonal polynomial and an orthogonal array, experiments will be orthogonal even for a multi-level array.

In general, an orthogonal design is very efficient for functions having a low degree and those in which the variables do not have interactions. It is disadvantageous, however, in that for functions having a high degree and those in which the variables have interactions, the number of parameter sets (number of experiments) increases and the regression model is limited to an orthogonal polynomial. This, however, enables parameter setting merely by selecting an orthogonal array and, therefore, offers excellent convenience.

3. Application to FOA sizing optimization

3. 1 Application models and design variables

In FOA, sizing optimization can be said to play the role of determining the optimal values of the size of a shape as determined with a method such as a topology optimization calculation method, or those for an existing design example, under those conditions in which constraints for practical use are considered. For this reason, sizing optimization provides information on the degree of effect on performance that can be used to determine which of several design parameters to select.

Thus, in this research, we adopted the response surface methodology,[5] given the flexibility

Table 1 L8(2^7) orthogonal arrays.

No.	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

(convenience) and speed of parameter setting. Here, we evaluate the applicability of this technique, using the problem shown in **Fig. 3** as an example, which consists of six beams.

We assume that the structure in the example is symmetrical, as shown in Fig. 3, and that the constituent beams are round bars. We also assume that the design variables are the horizontal position of the center point and the diameter of the cross-bars. The position of the center point is normalized with its ratio to the entire length, and the diameter is normalized with its ratio to the diameter of the other members. We assume two object functions, one for maximizing the stiffness at the load point and the other for minimizing the weight. **Table 2** lists the conditions for optimization.

3. 2 Design of experiments and response surface methodology

For this example, we use a three-level orthogonal array. For the design variables, we use an L9 orthogonal array because it has two variables. In **Table 3**, design variables are allocated to the L9 orthogonal array. In **Table 4**, the stiffness and mass values resulting from analysis in accordance with the orthogonal array are given, as normalized with their respective means. The objective functions are turned into a single object function as a weighting problem, since the property values of interest are multiple purposes of stiffness and mass.

Figure 4 shows the mean value plots of the design variables after conducting a variance analysis. From this graph, we can easily determine the superiority

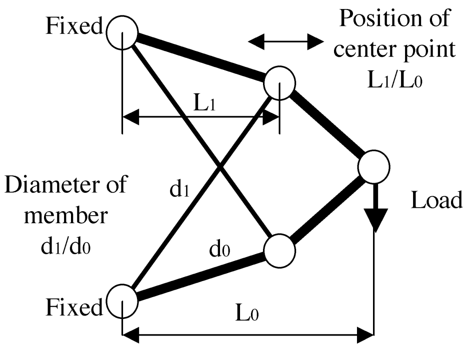

Fig. 3 Example of shape optimization.

Table 2 Condition of optimization.

		Variable	Minimum	Maximum
Design variable		Position of center point	0.1	0.9
		Diameter of member	0.1	0.9
Objective function		Stiffness	Maximize	
		Weight	Minimize	

Table 3 L9 orthogonal arrays.

	Position of center point	Diameter of member
1	0.1	0.1
2	0.1	0.5
3	0.1	0.9
4	0.5	0.1
5	0.5	0.5
6	0.5	0.9
7	0.9	0.1
8	0.9	0.5
9	0.9	0.9

Table 4 Result & Objective function.

	Stiffness	Total mass	Objective function
1	0.7958	4.1996	4.9954
2	0.9572	2.1613	3.1185
3	1.1186	1.649	2.7676
4	0.7605	0.1685	0.929
5	0.9528	0.1684	1.1213
6	1.1451	0.1684	1.3135
7	0.84	0.2648	1.1048
8	1.09	0.1242	1.2142
9	1.3399	0.0958	1.4357

and the degree of influence (sensitivity) of each design variable. In this example, we can determine that the degree of influence of the position of the center point is high and that the changes can be represented by a quadratic function.

Figure 5 shows the distribution of the response surface for the object function in this example. We can find the optimal solution by solving this function as a minimization problem using an appropriate method such as linear programming.

Figure 6 shows the shape after optimization. **Table 5** lists the optimal values of the design variables. The determined optimal solutions produce a shape in which the two beams form an

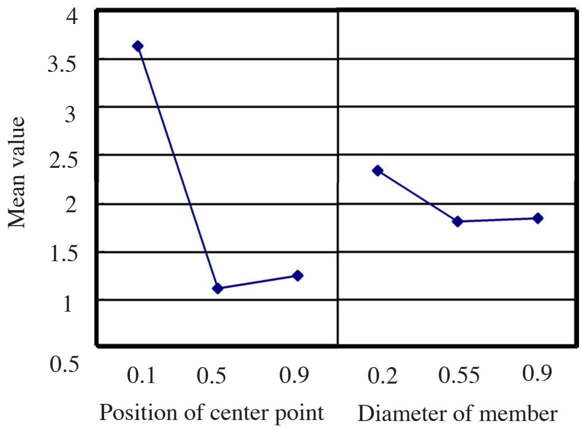

Fig. 4 Mean value plots.

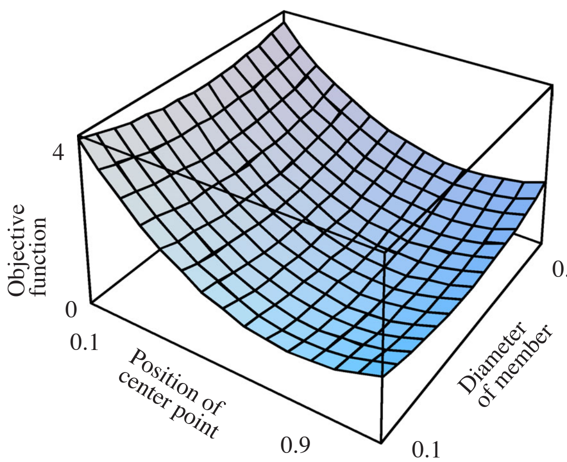

Fig. 5 Response surface.

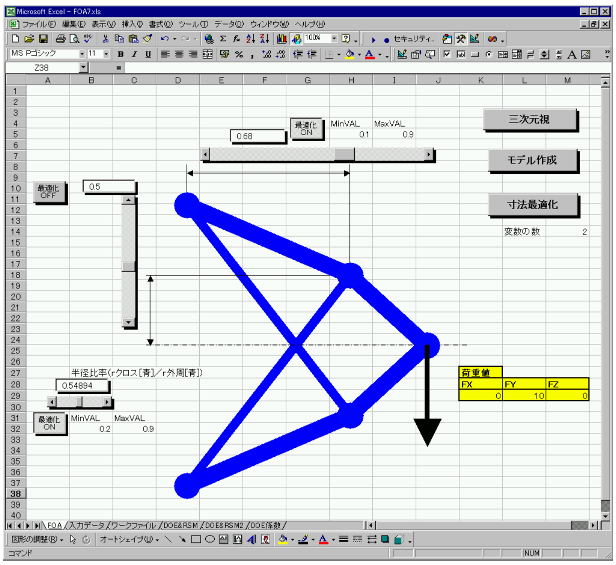

Fig. 6 Optimal design.

Table 5 Optimal design value.

Variable	Optimal value
Position of center point	0.679
Diameter of member	0.549

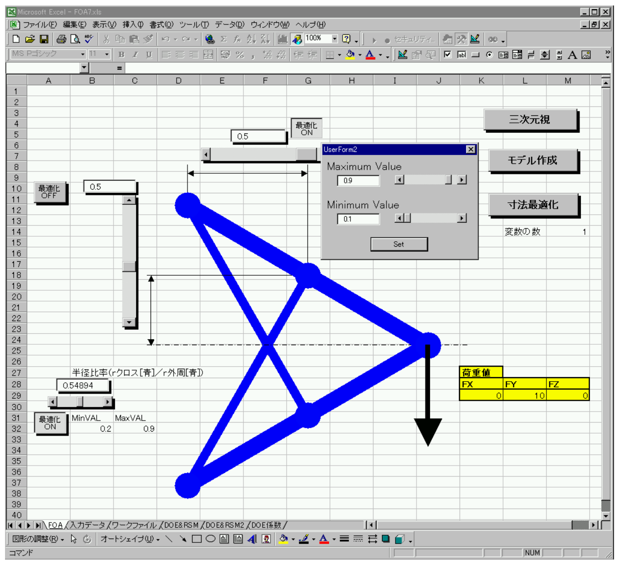

Fig. 7 Shape optimization sheet of FOA.

angle of about 90 degrees at the load point and the intersection. The direction determined by this angle is the direction of principal stress for shear load, while the axial direction with the highest beam stiffness is identical to this direction. These results correspond well to those of topology optimization[8] using planar elements. As described above, the optimal solutions indicate valid results, thus proving the validity of this technique.

3. 3 FOA program

The FOA program that uses this technique is as shown in **Fig. 7** when applied to the example described above. Merely by selecting the design variable to be optimized and defining the upper and lower limits, FOA automatically prepares a parameter set such as an orthogonal array. Then, analysis is performed a specified number of times, and from the results, optimization is automatically performed with the response surface methodology. The series of calculations is executed by Microsoft Excel macros, allowing the designer to obtain an optimal shape simply by manipulating a sheet (Fig. 6).

4. Conclusions

We have introduced an optimization technique that applies the response surface methodology to sizing optimization in FOA. The response surface methodology was outlined, and the effectiveness of this technique was confirmed using a two-dimensional beam problem as an example.

In FOA, it is possible to modify a design candidates in accordance with an actual design procedure and, in addition to this, design engineers can use optimization functions to create an initial design candidate and to perform sizing optimization while considering practicality.

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