

## **Prediction of Structural and Kinematic Coupled Vibration on Internal Combustion Engine**

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### **Abstract**

Calculational methods to predict the vibration of an internal combustion engine have become more important as demands for matching the lighter weight of automobiles with low vibration levels and for reducing the development period have been increasing. The former studies of engine vibration are categorized into two groups. One deals with structural vibration (elastic vibration) in relatively higher frequencies. The other deals with kinematic vibration (rigid body motion) in low frequencies. Recently, it is becoming more important to analyze structural and kinematic coupled vibration in middle range frequencies. In this study, we developed an

engine vibration analysis system (EVAS) for a computer aided design software, which can comprehensively analyze structural and kinematic coupled vibration in a wide range. A new formulation using a local observer frame and eigenmodes was proposed to efficiently calculate the structural and kinematic coupled vibration of the moving elastic body. In addition, some types of force elements were developed to express the transmitting force from a body to another body or the boundary conditions. The developed program was applied to a real engine model and verified by experiment under running conditions.

<b>Keywords</b>
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Internal combustion engine, Structural vibration, Kinematics, CAE, Flexible multibody dynamics, Numerical integration method, Local observer frame, Eigenmode, Bearing load

## 1. Introduction

As the demands for low fuel consumption in automobile engines and the reduction of engine development period increase, calculational methods of engine vibration are becoming more important in structural design to match lighter weight with lower vibration. Previous studies of engine vibration analysis are categorized into two groups. One deals with structural vibration (elastic vibration) in relatively higher frequencies. Generally, this is calculated by a finite element analysis solver such as NASTRAN<sup>1)</sup> under the assumption that the vibration is infinitesimal. In this case, the coupled vibration between the crankshaft and cylinder block through main bearings is analyzed using a rotational coordinate system for the crankshaft and an inertial coordinate system for the cylinder block<sup>2, 3)</sup>. The other deals with kinematic vibration (rigid body motion) in very low frequencies, which is solved by multibody dynamics theory using differential algebraic equations (DAE) such as DADS<sup>4)</sup>.

On the other hand, to reduce the vibration in a middle frequency range, it is also becoming more important to analyze the structural and kinematic coupled vibration. In this frequency range, the crankshaft rotates with elastic vibration that includes many torsional and bending modes. The cylinder block vibrates elastically with large rigid motion caused by the combustion force of the cylinder and the inertia force of the piston-conrod system. In addition, as the vibration of the crankshaft is transmitted to the cylinder block through the oil film bearings, the nonlinear dynamic characteristics of the oil film significantly influences the interaction between the crankshaft and the cylinder block.

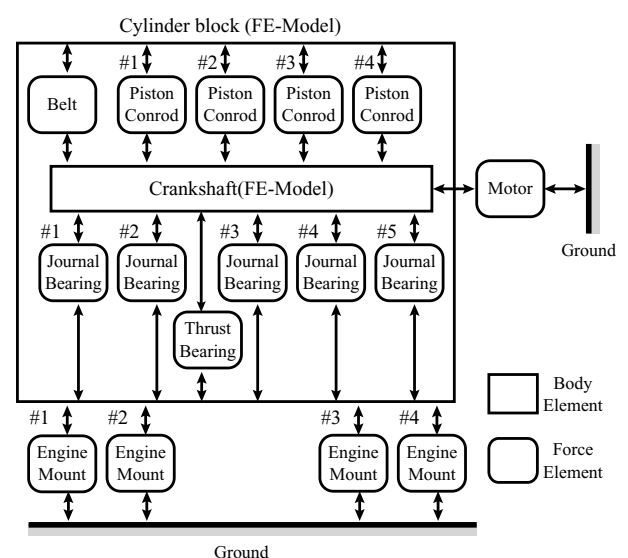
From these features, it is impossible to solve the structural and kinematic coupled vibration by the linear finite element analysis method. On the other hand, a multibody solver like DADS deals with structural and kinematic coupled vibration by adding the elastic mode to the rigid body. However, this method is not always practical because DAE includes three kinds of coordinates (Cartesian coordinates, Euler parameters and modal parameters)<sup>5)</sup> and many complicated coupling terms are derived from the coupling between the rigid

body motion and the elastic vibration. In addition, the simple constraint equation of DAE, such as for a revolutionary joint, is not sufficient to express the nonlinear characteristics of the oil film bearing.

From this background, in the former study<sup>6)</sup> we proposed a new formulation that is more practical than DAE for the structural and kinematic coupled vibration. In this study, we developed the engine vibration analysis system (EVAS) based on its formulation for a computer aided design tool. In EVAS, the important engine components such as the crankshaft and cylinder block are modeled by body elements whose motion and vibration are solved by its formulation. In addition, several types of force elements are introduced to express the boundary conditions of each component and the interaction between components. The developed system was applied to an inline 4-cylinder gasoline engine and verified by experiment under running conditions.

## 2. Outline of Engine Vibration Analysis System (EVAS)

**Figure 1** shows the system schematic of the inline 4-cylinder engine as shown in **Fig. 2**. The system is composed of body elements and force elements. The body element is expressed by a finite element model and is used for the moving elastic body such as a crankshaft and a cylinder block. The force



**Fig. 1** Schematic of EVAS.

elements express the boundary condition of body elements and the interaction between body elements. According to the connecting condition and function, the force elements are categorized as piston-conrod element, journal bearing element, motor element, engine mount element, thrust bearing element and belt element.

### 3. Formulation of EVAS

#### 3.1 Body element

The motion and vibration of a body element are calculated by the equation of motion formulated using a local observer frame (LOF) and both rigid and elastic modes of the body. The equations of motion are written by modal coordinate  $q_k$  as follows.

$$\begin{aligned} \ddot{q}_k + \sum_{i=1}^N m_i \phi_{ik}^T A^T a + \sum_{i=1}^N m_i \phi_{ik}^T A^T (\tilde{\alpha} + \tilde{\omega} \tilde{\omega}) A \left( s'_i + \sum_{j=1}^n \phi_{ij} q_j \right) \\ + 2 \sum_{i=1}^N m_i \phi_{ik}^T A^T \tilde{\omega} A \sum_{j=1}^n \phi_{ij} \dot{q}_j + \lambda_k q_k + c_k \dot{q}_k = \sum_{i=1}^N \phi_{ik}^T A^T F_i \end{aligned} \quad (1)$$

Refer to the former report<sup>6)</sup> for details about the derivation of Eq.(1) and the LOF. In Eq.(1),  $a$  is the acceleration of the LOF,  $\tilde{\omega}$  is the skew-symmetric matrix of the angular velocity  $\omega$ ,  $\tilde{\alpha}$  is the angular acceleration ( $=\dot{\tilde{\omega}}$ ), and  $A$  is the direction cosine matrix. The eigenvalue  $\lambda_k$  and the eigenvector  $\phi_{ik}$  of the body are obtained by modal analysis of a finite element solver. The modal proportional viscous damping coefficient  $c_k$  is estimated by experimental modal analysis. Eq.(1) is calculated by time integration method, as mentioned later.

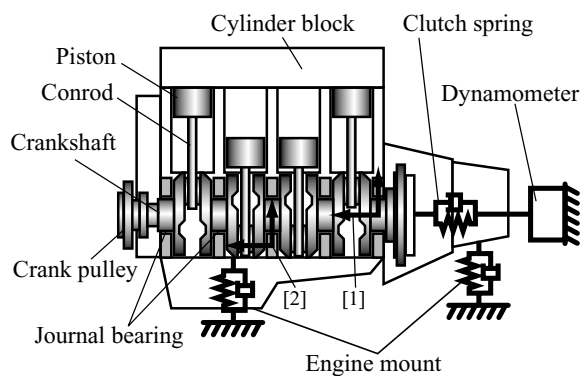


Fig. 2 Model of inline 4-cylinder engine.

The LOF observes the motion of the body locally while being located in the vicinity of the body, as shown in Fig. 3. At each time step in the calculation, the LOF is updated appropriately so that the modal coordinate  $q_k$  should be infinitesimal in Eq.(1). In Fig. 2, the LOF for the crankshaft is coordinate [1], and that of the cylinder block is coordinate [2]. Though the LOF can be set at the arbitrary position of the body element, for a rotating body such as a crankshaft the LOF should be attached to the rotating axis for simplification of the calculation.

#### 3.2 Force element

The force elements acquire the state values such as displacement and velocity of body elements at each time step, and calculate the force  $F_i$  in Eq.(1) using the state values. The calculated force is given to the body elements. The calculational method differs depending on the kind of force element.

Figure 4 shows a piston-conrod element. This element generates the combustion force  $F_c$ , the piston side force  $F_s$ , the friction force  $F_f$  between a piston and cylinder, and the crankpin input force  $F_{py}$  and  $F_{pz}$  according to the crank angle  $\theta$ . The combustion force  $F_c$  is calculated by the measured or predicted cylinder gas pressure. From the combustion force, other forces are calculated by kinematic method. For example, the piston side force  $F_s$  is calculated by the following equation using the piston mass  $m_p$  and the equivalent reciprocating mass  $m_1$  of the conrod.

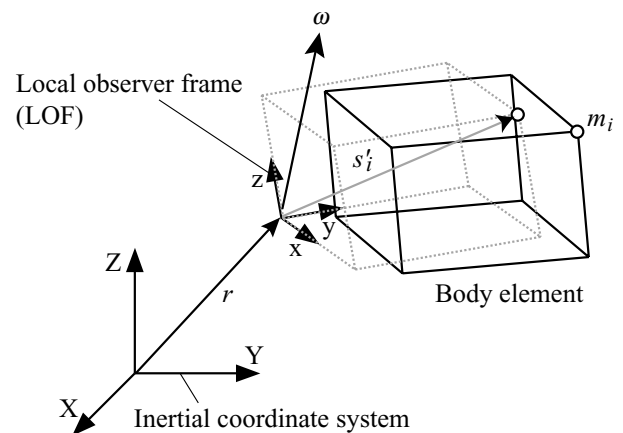
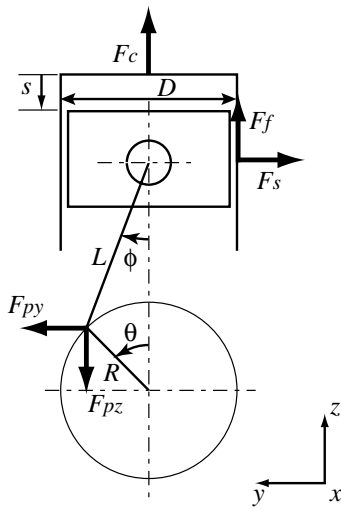


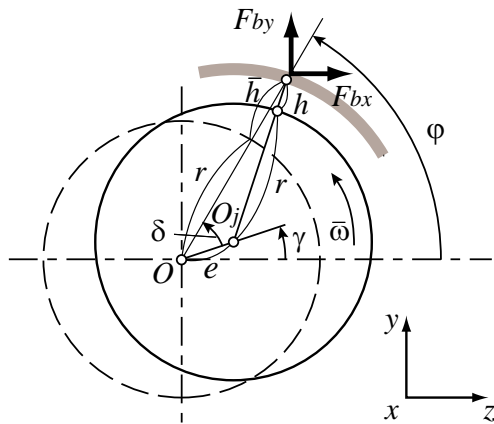
Fig. 3 Local observer frame.

$$F_s = - \left\{ F_c - R \dot{\theta}^2 (m_p + m_1) \left( \cos \theta + \frac{R}{L} \cos 2\theta \right) \right\} \tan \phi \quad (2)$$

**Figure 5** and **Figure 6** show the journal bearing element. This element generates the oil film force  $F_b$  and moment  $M_b$  from the eccentricity  $e$  of the journal center, the eccentricity velocity  $\dot{e}$ , the attitude angle  $\gamma$  and the attitude angular velocity  $\dot{\gamma}$  based on hydrodynamic lubrication theory. The oil film pressure  $p$  is approximately calculated using the next equation from Reynold's equation<sup>7)</sup>.



**Fig. 4** Piston-conrod element.

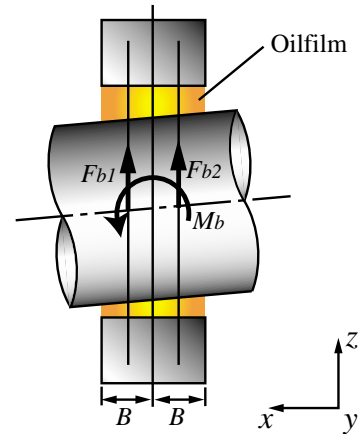


**Fig. 5** Journal bearing element (cross section).

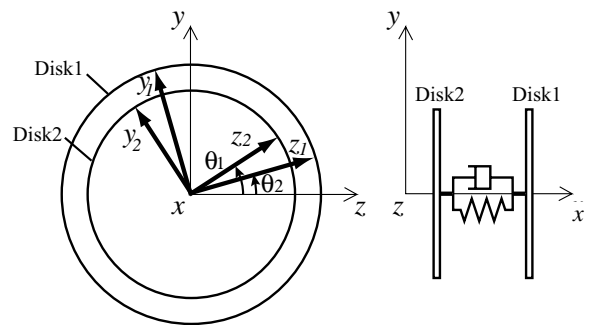
$$p = \frac{6\eta x (x-B)}{h^3} \left\{ \frac{\bar{\omega}}{2} \frac{\partial \bar{h}}{\partial \phi} + \left( \frac{\bar{\omega}}{2} - \dot{\gamma} \right) e \sin \delta - \dot{e} \cos \delta \right\} \quad (3)$$

where  $\eta$  is the viscosity coefficient of the oil,  $\bar{h}$  is the radial clearance when the journal center is at the bearing center,  $h$  is the radial clearance when the journal center is eccentric, and  $\bar{\omega}$  is the spin angular velocity of the journal. The oil film force  $F_{bx}$  and  $F_{by}$  are calculated by the integration of Eq.(3) in the bearing surface where the pressure  $p$  becomes positive. In addition, two journal elements are arranged as shown in Fig. 6 to express the moment  $M_b$  by the force  $F_{b1}$  and  $F_{b2}$ .

**Figure 7** shows the motor element which simulates the power transmission from the dynamometer to the clutch in engine bench tests. Disk 1 is a rotational



**Fig. 6** Journal bearing element (side view).



**Fig. 7** Motor element.

driver and it rotates as

$$\theta_1 = \omega t + \theta_0 \quad (4)$$

where  $\omega$  is the angular velocity of disk 1, and  $\theta_0$  is the initial phase. Disk 2 expresses the clutch. The torque  $T_r$  acting on the clutch is calculated by the rotational spring coefficient  $K_r$  and damping coefficient  $C_r$  which simulate a torsional clutch spring as follows.

$$T = -K_r(\theta_2 - \theta_1) - C_r(\dot{\theta}_2 - \dot{\theta}_1) \quad (5)$$

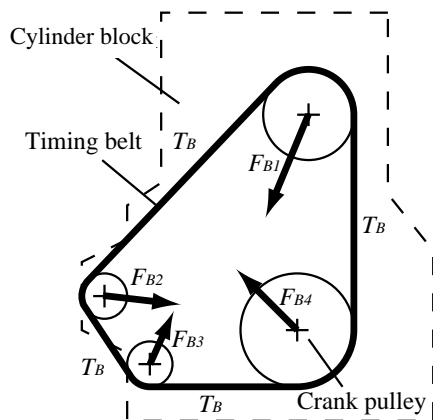
The engine mount element expresses the support force of engine mounts. The thrust bearing element expresses the constraint of the crankshaft in the axial direction. These elements are composed of a nonlinear spring and damper.

The belt element expresses the timing belt which connects a crank pulley and accessory pulleys as shown in **Fig. 8**. Here, the forces  $F_{B1}$ ,  $F_{B2}$ ,  $F_{B3}$ ,  $F_{B4}$  acting on each pulley center are calculated on the assumption that the belt tension  $T_B$  is constant.

#### 4. Numerical calculation of EVAS

In EVAS, the vibration response of the engine system is calculated by a time integration method. **Figure 9** shows the entire flow of the calculation.

In Step <1> in Fig. 9, the eigenvalue  $\lambda_j$  and eigenvector  $\phi_{ij}$ , calculated beforehand by a finite element solver, are set to the body elements. In Step <2>, the time is set up in the start time  $sT$  and the



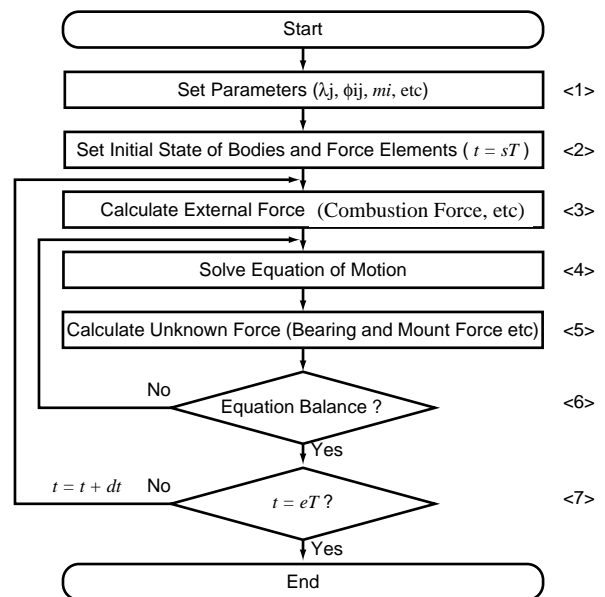
**Fig. 8** Belt element.

body elements, and the force elements are set to the initial condition.

Steps <3> to <6> are the procedure series for calculating the response at the time  $t$ . In Step <3> the forces determined explicitly by the function of time are calculated, such as the combustion force  $F_c$  and the crankpin input  $F_{py}$  and  $F_{pz}$ . In Step <4>, the response of each body element is calculated according to the acting force. In Step <5>, each force element acquires the state values of the body elements and calculates provisionally the unknown forces such as the journal bearing force  $F_{by}$  and  $F_{bz}$ , the driving torque  $T_r$  of the motor element and the support force of the mount element. The time integration of Step <4> is performed by a linear acceleration method. The convergence calculation is carried out by the numerical feedback loop from steps <4> to <6> as the balance error of Eq.(1) becomes 0.1% or less at the time  $t$ . If the calculation converges, the time  $t$  is advanced  $dt$  and the step returns to <3>. Below, steps <3> to <7> are repeated to the end time  $eT$ .

#### 5. Results of calculation and experimental verification

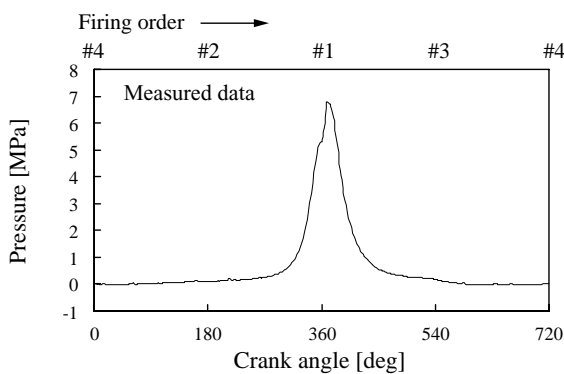
The exercise calculations for experimental verification were performed on an in-line 4-cylinder



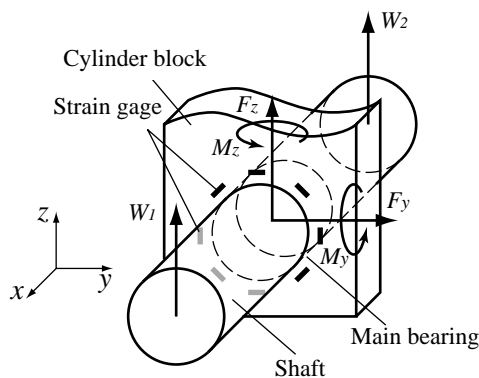
**Fig. 9** Flowchart of calculation.

diesel engine (4100cc). The running condition was 3000rpm full load. **Figure 10** shows the measured cylinder gas pressure used for the calculation. The 6 rigid modes and elastic modes below 2.5 kHz were used for the body element of the crankshaft and cylinder block. The crankshaft had 42 elastic modes and the cylinder block had 38 elastic modes.

The main bearing load was measured to verify the calculational results. The main bearing load corresponds to the oil film force of the journal bearing elements. It is influenced by the structural and kinematic coupled vibration, and is the main excitation force of the cylinder block. Therefore, it is an important factor in the structural design for reducing the cylinder block vibration. **Figure 11** shows the measurement method of the main bearing



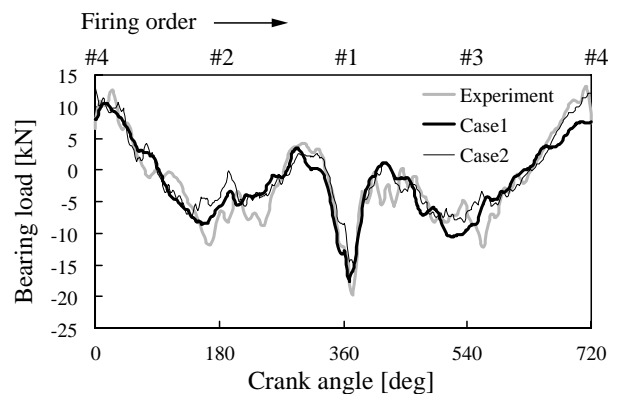
**Fig. 10** Cylinder gas pressure (#1 cylinder).



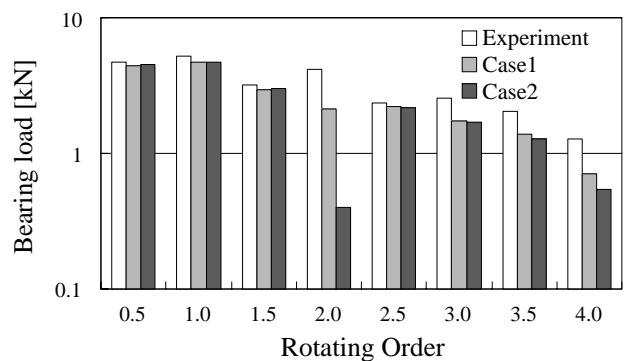
**Fig. 11** Measurement of bearing load.

load. The main bearing load under running conditions was estimated from the strain distribution around the main bearing. The relation matrix between the strain and bearing load was measured by static force tests beforehand. Refer to the former report<sup>8)</sup> for details of the measurement method.

**Figure 12** and **Figure 13** show the time history and frequency spectrum of the main bearing load. Case 1 is the calculational result for when the cylinder block is softly supported by a rubber mount, as with a real engine system. Case 2 is the calculational results for when the cylinder block is rigidly supported on the ground. It seems that the results of Case 1, Case 2 and the experiment agree with each other in Fig. 12. However, in the frequency analysis results of Fig. 13, a large difference appears in the 2nd



**Fig. 12** Bearing load.  
(#1 main bearing, vertical direction)



**Fig. 13** Spectrum of bearing load.  
(#1 main bearing, vertical direction)

rotating order component, and Case 1 corresponds to the experiment results better than Case 2.

The difference in Fig. 13 is due to the rigid motion of the whole engine. In the in-line 4-cylinder engine, the large rigid motions of the whole engine in the vertical and rolling direction occur with the 2nd rotating order due to the kinematic imbalance force of the piston-conrod system. This rigid motion influences the main bearing load. In Case 2, this influence was not considered because the rigid motion was fixed by the boundary condition. From these results, it was found that the coupling analysis of structural vibration and kinematic motion developed in this study was necessary to precisely predict the vibration transmission in the engine.

## 6. Conclusion

The engine vibration analysis system (EVAS) was developed by using the former proposed formulation for the moving flexible body and by introducing several kinds of force elements to express the boundary condition and interaction of the bodies. As the results of the experimental verification with the in-line 4-cylinder engine, it was found that EVAS could precisely predict the structural and kinematic coupled vibration. The influence of the rigid motion of the whole engine on the main bearing load was clarified. In addition, we also mention that the vibration of the cylinder block and mount was verified by the experiments on several kinds of engines. This system has already been used for the structural design modification of new types of engines in the TOYOTA Engine CAE system.

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