Research Report

Estimation of Tire-Road Friction by Tire Rotational Vibration Model

Takaji Umeno

Abstract

Tire-road friction is the most important piece of information used by active safety systems. However, most conventional friction estimation/detection methods are accurate only within the non-linear region. In addition, such methods require numerous sensors, such as yaw rate sensors, acceleration sensors, and steering angle sensors. In this paper, a friction estimation method using a tire vibration model is proposed. The method is based on the frequency characteristics of wheel speed vibration, which are related to tire-road friction. The recursive least squares and instrumental variable methods are applied for on-line estimation. The experimental results show that, when applied to a free-rolling tire, the proposed method detects friction change from dry asphalt to iced road while the vehicle travels at constant speed without braking, accelerating, or cornering.

Keywords Vibration, Tire-road friction, Estimation, Instrumental method, Safety system

1. Introduction

This paper presents an estimation method that can detect tire-road friction information during normal driving. In recent years, demand for automobile safety has increased, thereby promoting research and development of anti-lock brake systems (ABSs), collision avoidance systems, etc. Tire-road friction is the most important piece of information for these systems. However, most conventional friction estimation/detection methods are accurate only within the non-linear region. In addition, such methods require numerous sensors, such as yaw rate sensors, acceleration sensors, and steering angle sensors.

The proposed method is based on the frequency characteristics of wheel speed vibration; that is, strength of the resonance and frequency band. The recursive least squares and the instrumental variable methods are applied for on-line estimation. The main features of our method are (1) the friction condition can be estimated when the vehicle is still in the linear region, and (2) the only sensors required are wheel speed sensors, which are already provided for ABS. The experimental results show that when applied to a free-rolling tire, the proposed method detects the friction change from dry asphalt to iced road while the vehicle travles at constant speed without braking, accelerating, or cornering.

2. Tire vibration phenomenon and modeling

2. 1 Tire vibration phenomenon

Figure 1 shows the power spectrum density (PSD) of wheel angular velocity detected by an ABS wheel speed sensor at a constant vehicle speed of 60km/h. On the asphalt road, torsional resonance of the tire exhibits a large peak around the frequency of 40Hz [1]; however, no such clear peak is present on the low friction surface. The resonance characteristic of wheel angular velocity is considered to vary with tire-road friction, suggesting that tire-road friction can be estimated from the resonance characteristic of the wheel angular velocity (especially, the strength of the resonance) even when the vehicle is running straight at constant speed.

2. 2 Tire rotational vibration model

In this section, a tire vibration model for estimating tire-road friction from the resonance characteristic is introduced.

Figure 2 shows characteristics of the longitudinal force F_x with respect to slip velocity $S_V = V - \omega R$, where V is vehicle velocity, $\omega\$$ is wheel angle velocity, and R is tire radius. When F_x is depicted with respect to the slip ratio $S=(V - \omega R) / V$, the gradient of F_x at operating point S=0 is referred to as "braking stiffness". According to the experimental results of Gustafsson[2], the gradient of the friction coefficient (μ) with respect to S at μ=0 exhibits different values for asphalt and iced roads.

In this paper, Extended Braking Stiffness (XBS) is introduced as a replacement for brake stiffness. XBS is defined as the gradient of F_x at any point of

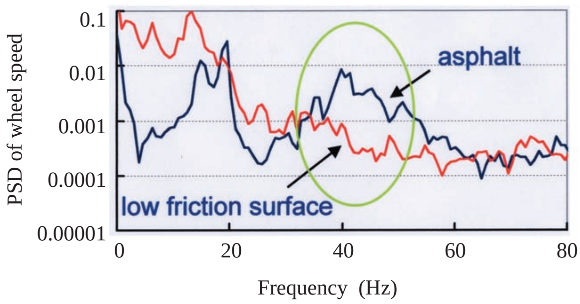

Fig. 1 PSD of wheel angular velocity.

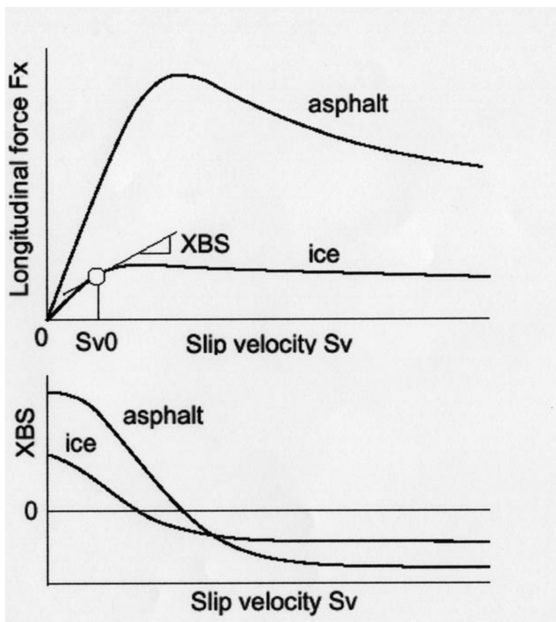

Fig. 2 Longitudinal force and extended braking stiffness.

slip velocity S_V, and is an important factor for estimating tire-road friction by the tire vibration model. The concept of XBS during braking has been reported by Sugai[3], whereby an antilock braking system was realized by estimating XBS at the point of maximum F_x. This research tries to estimate XBS during normal driving.

Figure 3 shows the tire vibration model considered in this paper, where, J_1 : moment of inertia at the rim side, J_2 : moment of inertia at the belt side, K : torsional spring constant of the tire, ω_1 : rotational speed at the rim side, ω_2 : rotational speed at the belt side, θ_s : torsional angle, T_L : longitudinal torque ($F_x R$) and T_d : disturbances from the road surface assumed to be white noise.

Motion equations in Fig. 3 are given as follows:

$$\begin{cases} J_1\dot{\omega}_1 = -K\theta_s \\ J_2\dot{\omega}_2 = K\theta_s + T_L + T_d \end{cases} \qquad \cdots\cdots(1)$$

In order to linearize system (1), a perturbation of T_L at an operating point $S_V = S_{V0}$ is derived:

$$\Delta T_L = \left.\frac{\partial T_L}{\partial S_V}\right|_{S_v = S_{v0}} \Delta S_v = \alpha R^2(\Delta V / R - \Delta\omega_2), \qquad \cdots\cdots(2)$$

where α is the XBS at $S_V = S_{V0}$. Under the assumption that $|\Delta\omega_2| >> |\Delta V / R|$ by virtue of the large difference between the inertia of the body and the wheel, Eq. (2) can be simplified to

$$\Delta T_L = -\alpha R^2 \Delta\omega_2 \qquad \cdots\cdots(3)$$

From the perturbation of system (1) and Eq. (2), a transfer function from the road disturbance ΔT_d to wheel speed $\Delta\omega_1$ is obtained as follows:

$$G(s) = \frac{K}{J_1\alpha R^2 s^2 + K(J_1 + J_2)s + K\alpha R^2} \qquad \cdots\cdots(4)$$

Figure 4 shows the frequency characteristic of $G(s)$. The resonance frequency f_r is determined by

$$f_r = \frac{1}{2\pi}\sqrt{\frac{K}{J_1}} \qquad \cdots\cdots(5)$$

and the strength of the resonance depends on XBSα, or tire-road friction.

3. Estimation method

In this section, the method for estimating the physical parameters of tire vibration model (5) is discussed. The frequency characteristic of the wheel angular velocity is assumed to be given by the output of the 2nd order system

$$G_2(s) = \frac{b_2}{s^2 + a_1 s + a_2} \qquad \cdots\cdots(6)$$

If the parameters a_1 and a_2 are determined, XBS α and resonance frequency f_r can be estimated by

$$\alpha = \frac{J_1 + J_2}{R^2}\frac{a_2}{a_1}, \quad f_r = \frac{1}{2\pi}\sqrt{a_2} \qquad \cdots\cdots(7)$$

In order to determine the parameters a_1 and a_2, parameter estimation for the continuous-time model[4] is used. The main feature of this method is direct estimation of physical parameters of the system. The continuous time model of Eq. (6) is given by

$$\Delta\omega_1(t) = \frac{b_2}{s^2 + a_1 s + a_2}\Delta T_d(t) \qquad \cdots\cdots(8)$$

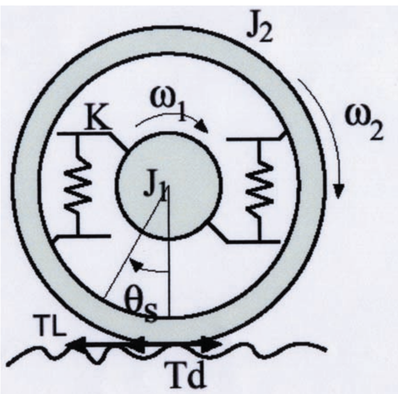

Fig. 3 Tire vibration model.

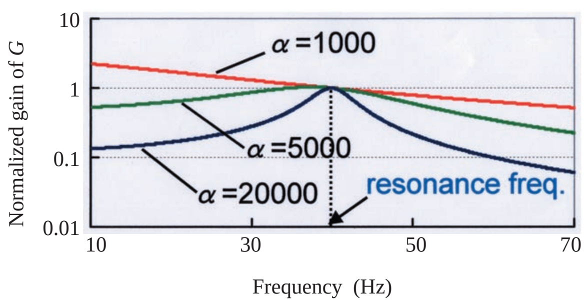

Fig. 4 Frequency characteristic of $G(j\omega)$.

which yields

$$s^2\Delta\omega_1(t) + a_1 s\Delta\omega_1(t) + a_2\Delta\omega_1(t) = b_2\Delta T_d(t) \quad \cdots(9)$$

By application of the Tustin transformation with sample period T_s to both sides of Eq. (9), the following equation is obtained.

$$\xi_{y0} = \zeta^T(k)\theta + r(k) \quad \cdots\cdots(10)$$

where

$$\zeta(k) = -\left[\xi_{y1}(k) \;\; \xi_{y2}(k)\right]^T,$$

$$\theta = -\left[a_1 \;\; a_2\right]^T, \; r(k) = \sum_{i=0}^{2} a_i \xi_{td}(k)$$

and ξ_{y1} and ξ_{y2} are derived from $\Delta\omega_1$ and ΔT_d, respectively.

One of basic algorithms for estimating θ of Eq. (10) is the recursive least squares (RLS) method:

$$\theta(N) = \theta(N-1) + h(N)\left[\xi_{y0}(N) - \zeta^T(N)\theta(N-1)\right]$$

$$h(N) = \frac{P(N-1)\zeta(N)}{\rho + \zeta^T(N)P(N-1)\zeta(N)},$$

$$P(N) = \frac{1}{\rho}\left\{I - h(N)\zeta^T(N)\right\}P(N-1)$$

where ρ is a forgetting factor.

Estimation of XBS α from the torsional resonance of the tire may require that the resonance component of the wheel angular velocity be extracted by a filter such as a bandpass filter. However, the assumption that the noise $r(k)$ has a white noise characteristic is no longer guaranteed. In this case, the instrumental variable (IV) method is useful and results in good estimation. The IV method is given as follows:

$$\theta(N) = \theta(N-1) + h(N)\left[\xi_{y0}(N) - \zeta^T(N)\theta(N-1)\right]$$

$$h(N) = \frac{P(N-1)\,m(N)}{\rho + \zeta^T(N)P(N-1)\,m(N)},$$

$$P(N) = \frac{1}{\rho}\left\{I - h(N)\zeta^T(N)\right\}P(N-1)$$

$m(k)$ is selected such that it does not correlate to $r(k)$. In this paper, L sample delay of $\zeta(k)$ is selected as IV.

4. Simulation results

Figure 5 shows a block diagram of a simulation for confirming the estimation methods. Parameters for the simulation are set as shown in **Table 1**. In the tire vibration model, all parameters other than α are fixed. The bandpass filter is used to extract the component of the torsional resonance of wheel angular velocity, because actual wheel angular velocity usually exhibits an unsprung resonance around 16Hz (see Fig. 1).

Figures 6 and **7** show simulation results obtained by the RLS and IV methods, respectively. As shown in Fig. 6, RLS fails in the estimation of both α and the resonance frequency f_r. On the other hand, the IV method can estimate α and f_r accurately. The bandpass filter makes the equation error $r(k)$ colored

Table 1 Parameters for simulation.

J_1	0.5	kgm^2	J_2	0.5	kgm^2
K	3.16×10^4	Nm/rad	R	0.3	m
T_s	0.005	s	L	3	
ρ	0.99		α	variable	

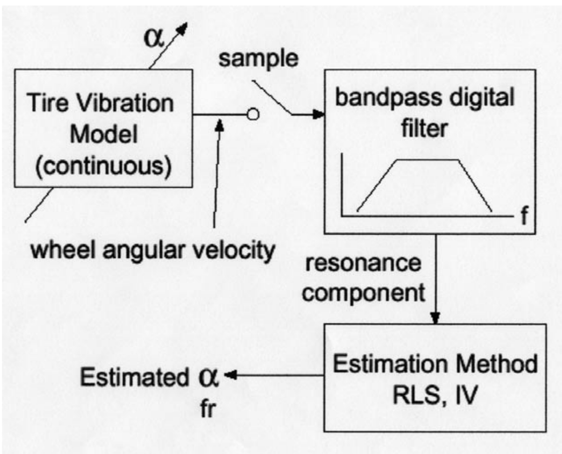

Fig. 5 Simulation blocks.

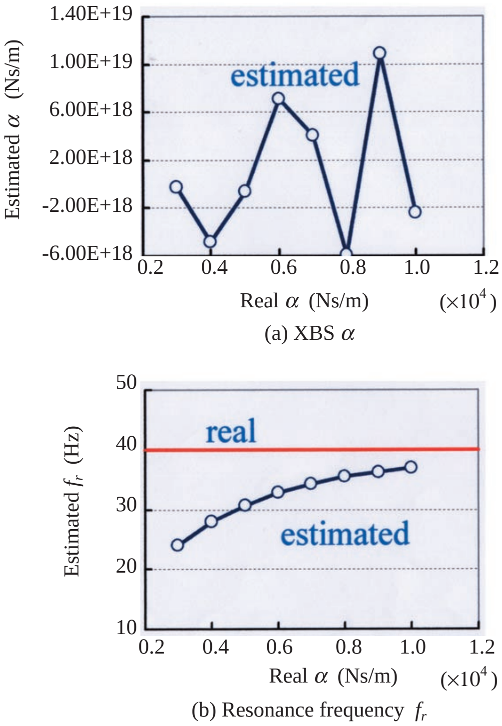

Fig. 6 Simulation results by RLS.

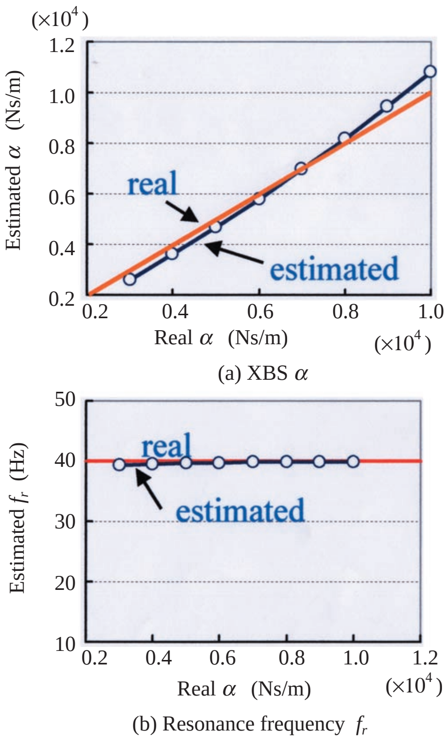

Fig. 7 Simulation results by IV method.

noise and results in failure of the RLS method. On the basis of these simulation results, the IV method is used to estimate α for experiments.

5. Experimental results

5. 1 Experimental system

The test vehicle is a rear-wheel-drive car equipped with ABS. The wheel speed sensor is composed of 48 serrations on the axle hub and a pickup coil. The wheel angular velocity is determined by shaping the sensor signal into a rectangular wave and measuring its period in the wheel speed processor (32bit microprocessor). Calculated wheel angular velocity of each wheel is sent through an ISA BUS to a PC, and XBS is estimated by the IV method.

5. 2 Friction change detection

Figure 8 shows the estimation result of the free rolling tire (right-front wheel) when the vehicle runs from dry asphalt to iced road at a constant speed of 60km/h. In this experiment, exact points of the road change were detected by the optical sensor. Two points of friction change are seen, at time 4.5 sec. (asphalt to ice) and 22.3 sec. (ice to asphalt). As can be seen, the estimated α varies immediately in response to the friction change.

5. 3 Hydroplaning detection

Figure 9 shows the estimation result for the hydroplaning test, and summarizes the estimated α for different vehicle speeds. The hydroplaning test road consists of submerged asphalt with rubber partitions. Hydroplaning occurs at speeds greater than 80km/h. The hydroplaning phenomenon is

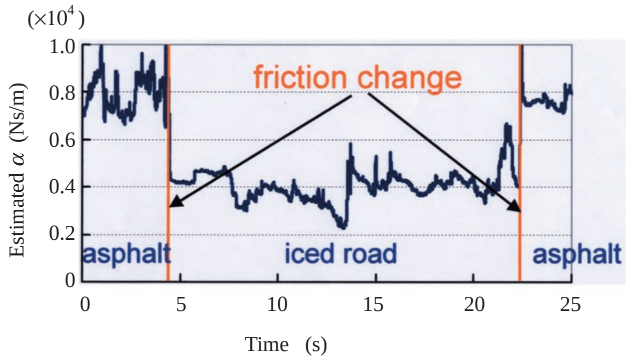

Fig. 8 Estimation result on the iced road.

observed when the tire begins to float on the wet surface by virtue of the dynamic pressure of water in the tire contact patch. At that time, the pure contact length of the tire decreases. Since XBS at $S_v = 0$ strongly depends on the contact length, these experimental results are considered to indicate the decrease in pure contact length by hydroplaning.

Figure 10 shows the time response of estimated α at a speed of 80km/h. The vehicle entered the submerged road at 1.0 sec, and the estimated α is found to drop at that time, whereby the hydroplaning phenomenon is detected in 0.2 sec.

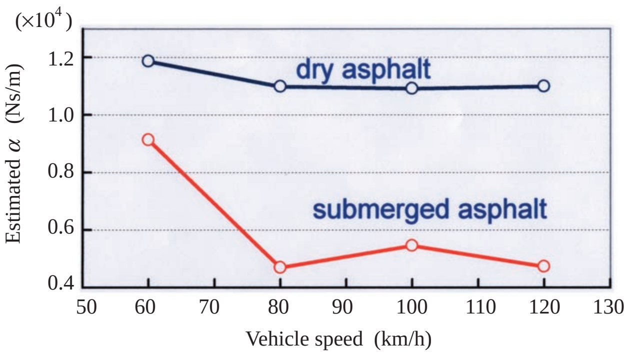

Fig. 9 Estimated α at submerged asphalt for each vehicle speed.

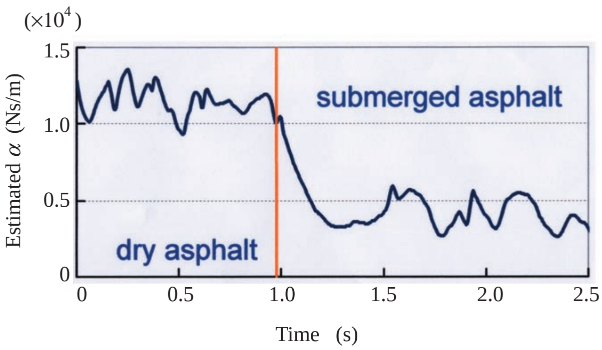

Fig. 10 Estimation result for hydroplaning test.

6. Conclusion

In this paper, the frequency characteristics of wheel speed vibration are shown to be related to tire-road friction. An estimation method based on these characteristics is proposed. The method is compared to experimental results, and friction change, including that resulting from hydroplaning, can be detected even at constant speed. Friction condition can be estimated when the vehicle is not necessarily performing any dynamic or handling maneuvers, and wheel speed sensors are the only sensors that are required.

References

1) Umeno, T. : "Observer based estimation of parameter variations and its application to tire pressure diagnosis", Proc. IFAC Workshop Advanced in Automotive Control, (1998), 23, IFAC

2) Gustafsson, F. : "Monitoring tire-road friction using tire wheel slip", IEEE Control Systems, (1998), 42

3) Sugai, M., et al. : "New control technique for maximizing braking force on antilock braking system", Vehicle System Dynamics, **32**(1999), 299

4) Young, P. : "Parameter estimation for continuous - time models - A survey", Automatica, **17**-1(1981), 23

(Report received on July 4, 2002)

Takaji Umeno

Year of birth : 1963

Division : Research-Domain 17

Research fields : Vehicle state estimation and control

Academic degree : Dr. Eng.

Academic society : IEE of Jpn., The Soc. of Instrum. Control Eng., IEEE.

Awards : IEEE/IES Outstanding Paper Award, 1994.
Robomec '94 Best Poster Award, 1995.
R&D 100 Award, 1997.