

**Research  
Report****Modeling of Tire Overturning Moment Characteristics and the Analysis of Their Influence on Vehicle Rollover Behavior**

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**Abstract**

A newly developed tire model for the Overturning Moment (OTM) characteristics and the analysis of the influence of OTM on vehicle rollover behavior are presented. The new OTM model was developed based on the so-called Magic Formula tire model. The concept of the new model involves identifying the difference between the simple model used previously and actual measurements to the newly defined functions. The new model agrees very well with the measured data over a wide range of tire

vertical loads, slip angles and camber angles.

The influence of tire OTM on the vehicle rollover behavior was also investigated by means of a full vehicle simulation in which a rather large steering angle was input. The results obtained from the vehicle models with three different tire models (model without OTM, simple model, and new model) were compared with experimental results. It was found that the calculated result obtained with the new OTM model agreed best with the experiment.

**Keywords**

Tire model, Overturning moment, Vehicle dynamics, Rollover, Simulation

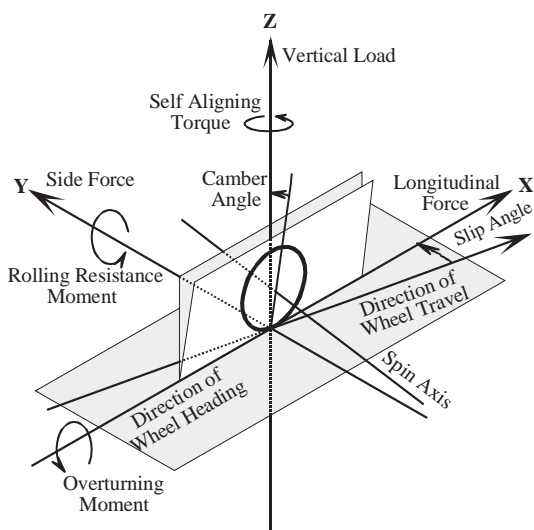
## 1. Introduction

It is well recognized that tires play an essential role in all aspects of vehicle behavior. Therefore, a huge number of studies to analyze and model tire properties have been undertaken to understand and simulate vehicle behavior. Among those studies, the tire longitudinal force, the side force and the aligning torque characteristics have been studied thoroughly from various points of view to improve vehicle dynamics analysis, but very few studies have considered the tire overturning moment (abbreviated to OTM, below) characteristics.

The authors also have been studying the modeling of the tire characteristics as they relate to improving the accuracy of vehicle dynamics simulations by using the so-called Magic Formula (abbreviated to MF, below) tire model.<sup>1-3)</sup> In this paper, a new tire OTM model is presented, which was developed as a final part of our studies relating to the modeling of tire steady-state characteristics. In addition, the influence of OTM on vehicle rollover behavior is analyzed by comparing the results obtained by experiment and those obtained by simulation.

## 2. Definition of tire coordinate system and cause of OTM generation

The coordinate system used for defining the tire force and moment characteristics is shown in **Fig. 1**. The OTM is the moment acting around the



**Fig. 1** The tire coordinate system.

longitudinal axis. The cause of the generation of OTM may be understood as the lateral shift of the acting center of the tire vertical load during vehicle cornering (see **Fig. 2**). The amount of that lateral shift is called a pneumatic scrub.

## 3. Comparison of simple OTM model with measurements

Considering the generation of OTM as noted above, the measured pneumatic scrub can be calculated using Eq. (1) from the measured OTM and the vertical load. On the other hand, the simple treatment of the pneumatic scrub (Eq. (2) below) has been proposed with the following assumptions, which is named the "Simple Model" in this study.

- 1) The tire contacts the road surface at a point.
- 2) The cross section of the tire tread is circular.
- 3) The tire contact point moves laterally under the tire side force and the lateral stiffness.

$$P_{s,m} = M_x / F_z \quad \dots\dots\dots (1)$$

$$P_{s,s} = 1 / K_L \cdot F_y - R_L \tan \gamma \quad \dots\dots\dots (2)$$

Here,

$F_y$  : Side force

$F_z$  : Vertical load

$K_L$  : Lateral stiffness

$M_x$  : Overturning moment

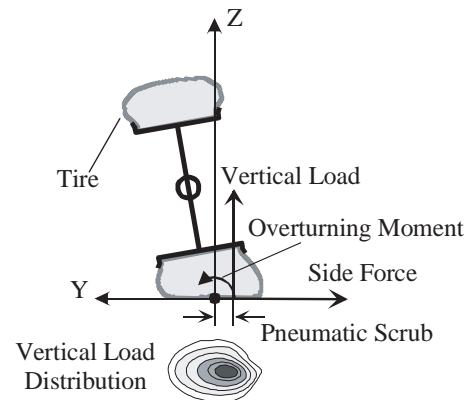
$P_{s,m}$  : Pneumatic scrub by direct measurement

$P_{s,s}$  : Pneumatic scrub by simple model

$R_L$  : Tire static radius

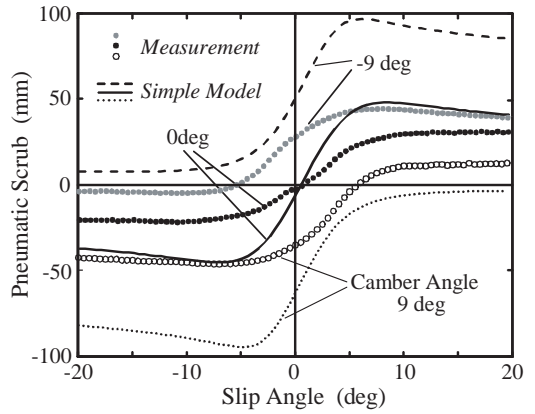
$\gamma$  : Camber angle

The two different pneumatic scrubs, namely, the measured value (Eq. (1)), and the value calculated

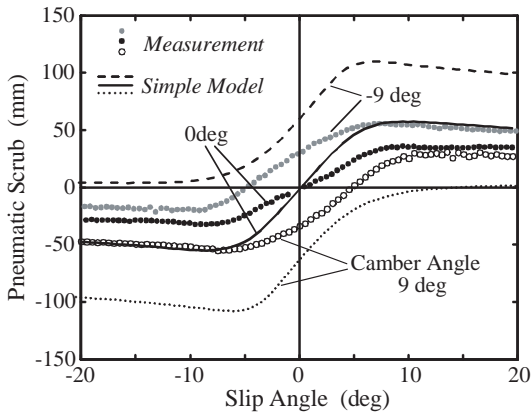


**Fig. 2** Schematic sketch of OTM generation.

from Eq. (2), were compared as shown in **Fig. 3**. The side force  $F_y$  in Eq. (2) was calculated by using the MF tire model,<sup>3)</sup> while the values of the lateral stiffness and the static radius of two tires are listed in **Table 1**. The tire force and moment characteristics were measured by using a flat belt



(a) Tire size : 215/70R16, Vertical load : 7.3 kN



(b) Tire size : 265/70R16, Vertical load : 9.8 kN

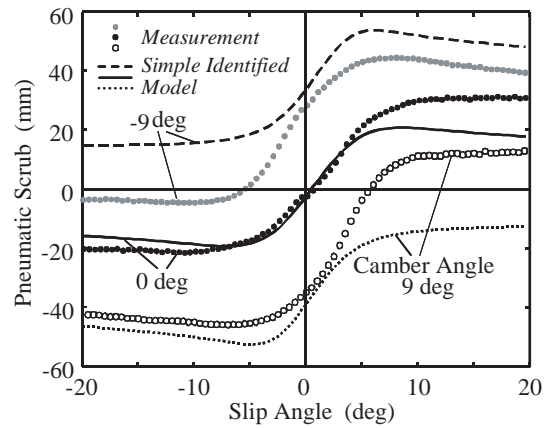
**Fig. 3** Comparison of pneumatic scrub between measurement and Simple Model.

**Table 1** Lateral stiffness and static radius.

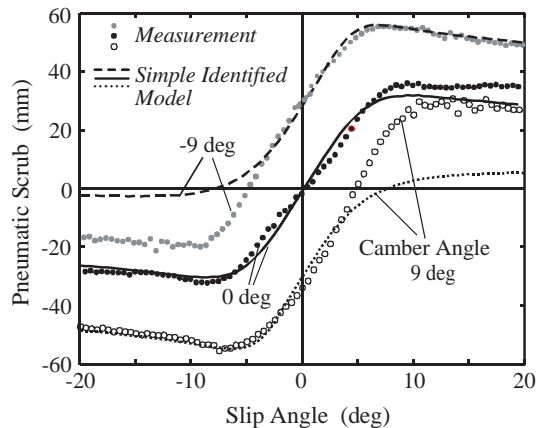
Tire	215/70R16	265/70R16
Lateral Stiffness (N/mm)	129	135
Static Radius (mm)	319	350

type tire test machine. From Fig. 3, we can see that there are rather large disagreements between the measured results and those calculated using the simple model, especially in the region of large slip angles and camber angles.

In the literature,<sup>4)</sup> a similar equation was used, but the parameters,  $K_L$  and  $R_L$ , were determined by identifying the measured data of OTM instead of the actual values, so the same trial was performed, as shown in **Fig. 4** (named the "Simple Identified Model" here). It is thought that the degree of agreement can be improved, but that disagreements will still exist in the region of large slip angles and camber angles. This shows that the simple treatment of OTM generation agrees poorly with the actual tire behavior, which may be caused by a change in the



(a) Tire size : 215/70R16, Vertical load : 7.3 kN



(b) Tire size : 265/70R16, Vertical load : 9.8 kN

**Fig. 4** Comparison of pneumatic scrub between measurement and Simple Identified Model.

vertical load distribution of the tire contact patch and from an error in the simple assumptions noted above. Then, the new tire OTM model was developed to obtain better agreement with the measurements.

#### 4. Development of new OTM model

##### 4.1 Model description

First, we observed the difference between the pneumatic scrub obtained from measurement and that obtained with the simple model, as shown in **Fig. 5**. This difference is called a "Residual Pneumatic Scrub" in this paper. The shape of a residual pneumatic scrub depending on the slip angle seems to be similar to that of the side force. Then, the new term for the residual pneumatic scrub,  $P_r$ , was added to Eq. (2) and the modified pneumatic scrub,  $P_s$ , was newly defined. For the residual pneumatic scrub, the same expression for the side force of MF<sup>3)</sup> was used as below because of the similarity in the shape. The following model is named the "New Model".

$$P_s = 1 / K_L \cdot F_y - R_L \tan \gamma - P_r \quad \dots\dots\dots (3)$$

$$P_r = D \sin$$

$$[C \arctan\{Bx - (E_0 + \Delta E \operatorname{sgn}(x))(Bx - \arctan Bx)\}] + S_v \quad \dots\dots\dots (4)$$

$$x = \alpha + S_h \quad \dots\dots\dots (5)$$

$$K_L = m_{18} \quad \dots\dots\dots (6)$$

$$R_L = m_{19} \quad \dots\dots\dots (7)$$

$$C = m_0 \quad \dots\dots\dots (8)$$

$$D = (m_1 F_z^2 + m_2 F_z)(1 - m_{15} \gamma^2) \quad \dots\dots\dots (9)$$

$$BCD = m_3 \sin [2 \arctan (F_z / m_4)](1 - m_5 / \gamma) \quad \dots\dots\dots (10)$$

$$B = BCD / CD \quad \dots\dots\dots (11)$$

$$E_0 = m_6 F_z^2 + m_7 F_z \quad \dots\dots\dots (12)$$

$$\Delta E = -(m_6 F_z^2 + m_7 F_z)(m_{16} \gamma + m_{17}) \operatorname{sgn}(\alpha + S_h) \quad \dots\dots\dots (13)$$

$$S_h = m_8 F_z^2 + m_9 F_z + m_{10} F_z \gamma \quad \dots\dots\dots (14)$$

$$S_v = m_{11} F_z^2 + m_{12} F_z + (m_{13} F_z^2 + m_{14} F_z) \gamma \quad \dots\dots\dots (15)$$

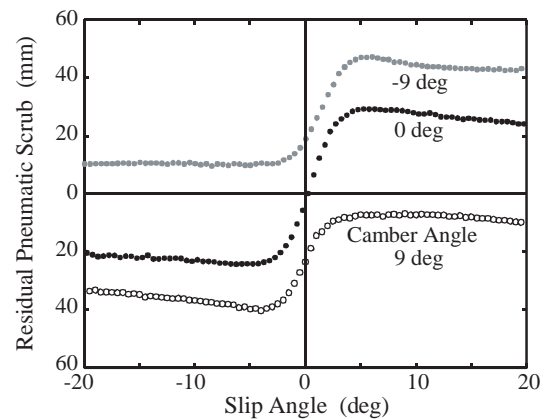
Where,  $\alpha$  is the slip angle, and  $B$  to  $S_v$  in Eqs. (4) and (5) are called the Magic Formula coefficients, which are functions of the vertical load and the camber angle. In Eqs. (6) to (15),  $m_0$  to  $m_{19}$  are called the Magic Formula parameters.

##### 4.2 Model identification procedure and results

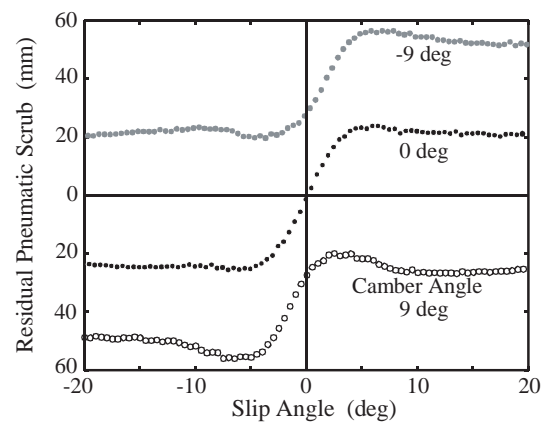
To successfully establish the MF tire model, the optimum values of the MF coefficients and parameters have to be determined. The authors have been developing a software system to identify those coefficients and parameters by using various types of measured data and identification methods (see Reference 3 for details).

The calculations for establishing the New OTM Model shown in **Fig. 6** were added to the former software. As shown in Fig. 6, the residual pneumatic scrub is calculated first, after which the MF coefficients and parameters are identified. Finally, by using the values obtained for the coefficients and parameters as initial values, all the coefficients and parameters are again simultaneously optimized by using the measured data.

Examples of calculated results obtained by the use



(a) Tire size : 215/70R16, Vertical load : 7.3 kN



(b) Tire size : 265/70R16, Vertical load : 9.8 kN

**Fig. 5** Residual Pneumatic Scrub.

of the New Model and the newly developed identification system are compared with the measured results in Fig. 7. It is clear that the New OTM Model agrees very well with the measurements over a wide range up to a large slip angle, camber angle and vertical load.

### 5. Influence of tire OTM on vehicle rollover behavior

Finally, to observe the influence of tire OTM on vehicle behavior, the vehicle rollover behavior was investigated, because it is easy to imagine that the influence of OTM appears more clearly with a large slip angle and vertical load by considering the cause of OTM generation (Fig. 2). To precisely analyze the influence of OTM on the vehicle rollover behavior, three tire models were used in the vehicle simulation and the calculated results were compared with vehicle experimental results. In vehicle dynamics simulations, the tire OTM has normally been disregarded (without OTM). The three tire models are,

- Without OTM
- Simple Model
- New Model

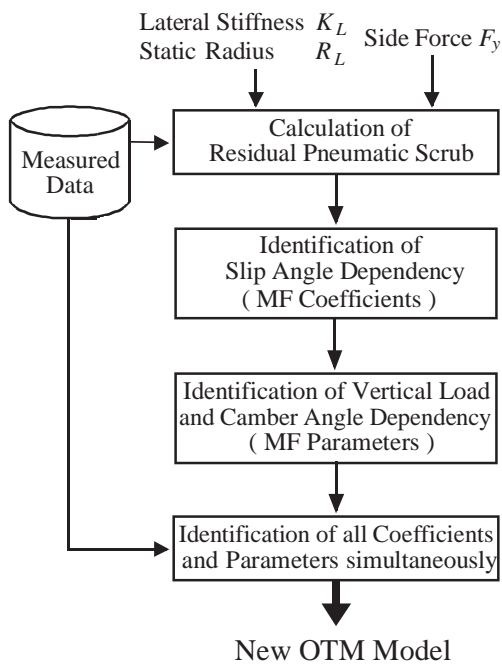
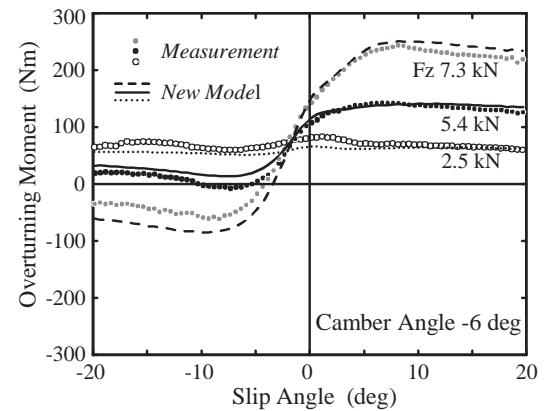
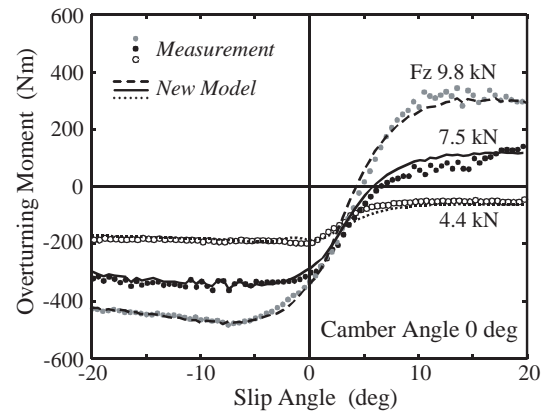


Fig. 6 Procedure of identification of New OTM Model.

For the vehicle description, a full vehicle model was developed using ADAMS with the parameter values for the vehicle used for the experiment. A



(a) Tire size : 215/70R16, Camber angle : -6 deg



(b) Tire size : 265/70R16, Camber angle : 9 deg

Fig. 7 Comparison of overturning moment between measurement and New Model.

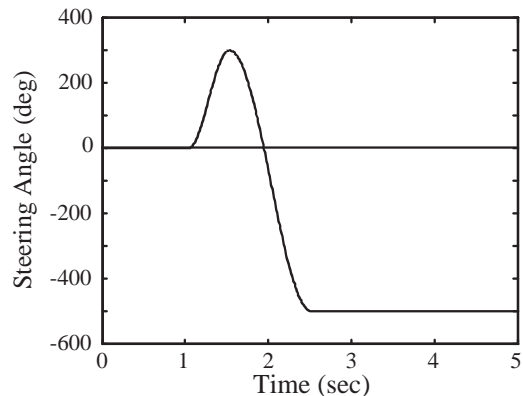
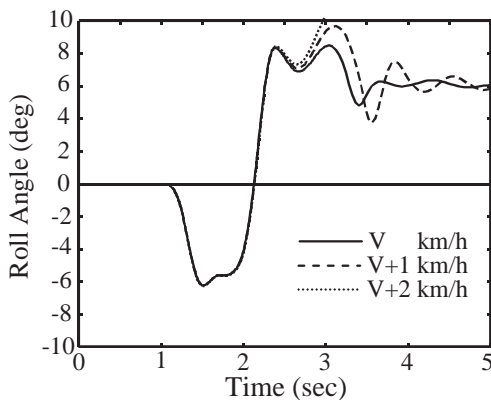


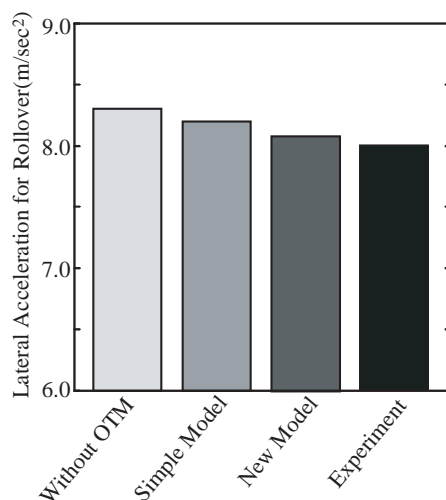
Fig. 8 The steering angle input for vehicle simulation.

relatively large steering angle input was applied as shown in **Fig. 8** for the simulation run, during which the vehicle forward speed was held constant.

We used three vehicle models, each with a different tire model. The simulation run was repeated starting from a low forward speed which was increased in steps of 1 km/h. As the vehicle behavior, the roll angle response was observed. As the forward speed increased, the roll angle response was altered as shown in **Fig. 9**. At the forward speed denoted as V+2 km/h in Fig. 9, the roll angle response diverged. Then, the maximum value of lateral acceleration in the simulation run of V+1 km/h was defined as the lateral acceleration for rollover. In **Fig. 10**, the lateral accelerations for



**Fig. 9** The response of roll angle.



**Fig. 10** Comparison of lateral acceleration for rollover between calculated results and experiment.

rollover are compared with the value obtained from the experiment, which was performed in the same way as the calculation. We can see that the effect of OTM on the tire reduces the lateral acceleration for rollover. We can also see that the result calculated with the New OTM Model agrees best with the result by experiment and that the inclusion of the tire OTM characteristics into the vehicle dynamics simulation improves the accuracy of the calculation.

## 6. Conclusion

The findings presented in this paper are summarized below.

(1) A new model for the tire overturning moment characteristics was developed based on the Magic Formula by adding a new term for the residual pneumatic scrub.

(2) To find the optimal values for the coefficients and parameters of the new OTM model, a calculation program was developed and added to the existing software system.

(3) It was found that the new OTM model agrees very well with the measured results.

(4) The influence of tire OTM on the vehicle rollover behavior was investigated. Three tire models were introduced to the vehicle simulation model and the calculated results were compared with those obtained by experiment. The effect of OTM on the tire reduces the lateral acceleration for rollover. The result calculated using the new OTM model agreed best with the experiment.

(5) The introduction of the new OTM model into vehicle dynamics simulation is thus highly recommended for improving the accuracy of the calculation.

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