

**Research  
Report**

## **Optimum Vehicle Dynamics Control Based on Tire Driving and Braking Forces**

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### **Abstract**

This paper discusses a method of controlling the tire force exerted by each wheel of a vehicle as a means of ensuring steerability and stability.

Conventional vehicle stability control systems generally rely on feedback of vehicle states such as the side slip angle and yaw rate to enable stabilization under a range of vehicle/road surface combinations. On a low-friction road surface that is incapable of applying sufficient force, however, it is unreasonable to expect a system to provide sufficient control upon the occurrence of undesirable vehicle behavior. The tire force has a non-linear saturating characteristic, the value of which varies with the vehicle state and road surface, making it difficult to determine the force to be generated by each tire to ensure the desired vehicle behavior.

This report proposes a method of distributing the target force and moment of a vehicle to each tire by considering variations in the tire force caused by changes in the tire's vertical load, the longitudinal slip, and so on. As a result, the proposed strategy achieves seamless behavior between the normal and the critical limit regions. And, we have confirmed that excellent levels of steerability and stability can be achieved, relative to conventional stability control systems, by simulating a slalom maneuver. Also, this strategy enables effective vehicle control in the face of unstable phenomena involving unbalanced lateral forces arising from changes in the vehicle behavior, such as closing the throttle during a turn maneuver.

**Keywords**

Vehicle dynamics control, Nonlinear optimum control, Tire model, Steerability, Stability

## 1. Introduction

Since the 1980s, various chassis active control systems have been developed to ensure vehicle stability. Direct-yaw moment control systems with active braking, as typified by VSC (Vehicle Stability Control) have realized good vehicle stability in the critical limit region.<sup>1-4)</sup> Those papers have mainly discussed control methods that feedback various aspects of the vehicle state, such as the side slip angle and yaw velocity.

Some subsequent proposals have introduced vehicle and tire models to estimate the vehicle state and calculate the yaw moment required to stabilize the vehicle.<sup>5-6)</sup> In earlier methods that handed over control of the vehicle immediately after detecting a vehicle skid, however, it was difficult to attain smooth control. Therefore, a method of directly controlling the force and moment of the vehicle was proposed.<sup>7)</sup> Meanwhile, the tire force exhibits non-linear saturating characteristics according to the vehicle state and road surface conditions. Additionally, these values change frequently. Therefore, it is not easy to determine the force that should be generated at each tire in order to obtain the target force and moment of the vehicle under any given driving situation. Hence, earlier systems used rule-based methods to calculate the longitudinal force at each wheel from the direct yaw moment.

Against this background, next-generation vehicle dynamics control should be able to provide seamless vehicle steerability and stability at any time while driving, integrated control systems to use most of the tire performance.

First, we propose the new VDM (Vehicle Dynamics Management) concept. Next, we suggest three key technologies that are central to realizing this concept. Finally, the performance of the proposed strategy is investigated by simulation.

## 2. Concept and key technologies of VDM

The ultimate goal of vehicle dynamics control is to produce vehicles that anyone can drive "safely," "pleasantly," and "as one wishes." In realizing this, we must embrace the key words of "seamlessly" and "anytime." To achieve this, we are proposing the VDM concept.

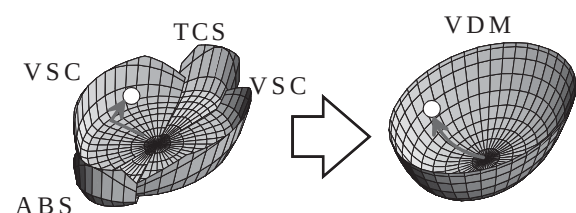
**Figure 1** illustrates the control provided by VDM. We refer to this illustration as the "Ball in a bowl." The ball corresponds to the vehicle state that is maintained within a conceptual "bowl" that corresponds to the control systems. Inside the bowl is the stable region, while outside it is the unstable region. In conventional systems, the wall is configured by several systems such as ABS, VSC, and TCS (Traction Control System), all of which are sheer just before facing an emergency situations. As a result, the vehicle motion can be stabilized, although non-smooth motion may sometimes occur. VDM, on the other hand, enables smoother behavior, because it involves the conventional control systems being restructured to form a continuous and smooth "wall." To realize this level of control, the following elements are important:

- Hierarchical algorithm involving the restructuring of the conventional control strategy
- Feedforward force and moment control for vehicle dynamics
- Nonlinear optimum distribution method that coordinates the operations of each actuator

Each of the above is discussed in the following chapters.

## 3. Hierarchical algorithms for VDM

Since vehicle control systems have continued to become more diversified, their algorithms need to be able to easily manage the cooperative control of multiple systems such as the drive train, braking and steering. Accordingly, it is important to be able to ensure the compatibility of the algorithms with different system configurations. To ensure the high level of performance, we require a model-based algorithm that uses feedforward control in addition to feedback control of the vehicle state.



**Fig. 1** The evolution of control by VDM.

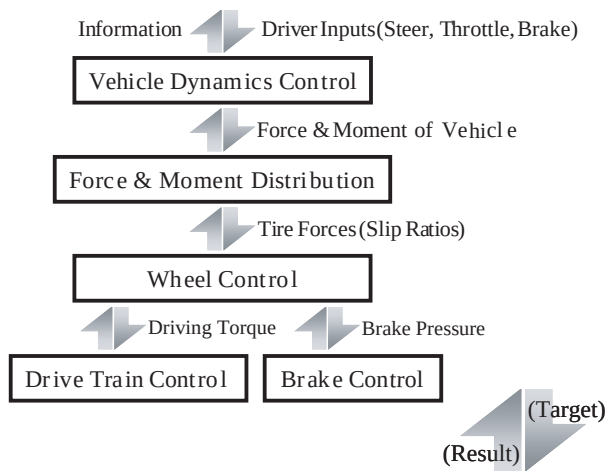
An HVDM (Hierarchical Vehicle Dynamics Management) algorithm has been proposed to satisfy the above requirements (**Fig. 2**). The HVDM algorithm consists of small systems, each connected hierarchically.

[Vehicle dynamics control]

This layer calculates the desired longitudinal and lateral forces, as well as the yaw moment of the vehicle. The forces and moment are determined as the vehicle assumes the desired motion, taking the driver's operations into account.

[Force and moment distribution]

This layer determines the distribution of the



**Fig. 2** Hierarchical vehicle dynamics management algorithm.

individual tire forces, with the total becoming the desired force and moment of the vehicle.

[Wheel control]

This layer calculates the target values for each actuator, such as the engine, braking, steering and so on. The target values are determined to generate the desired tire forces.

[Actuator control]

Each actuator system is controlled. The braking system, for example, uses actuated pressure valves to control the braking torque.

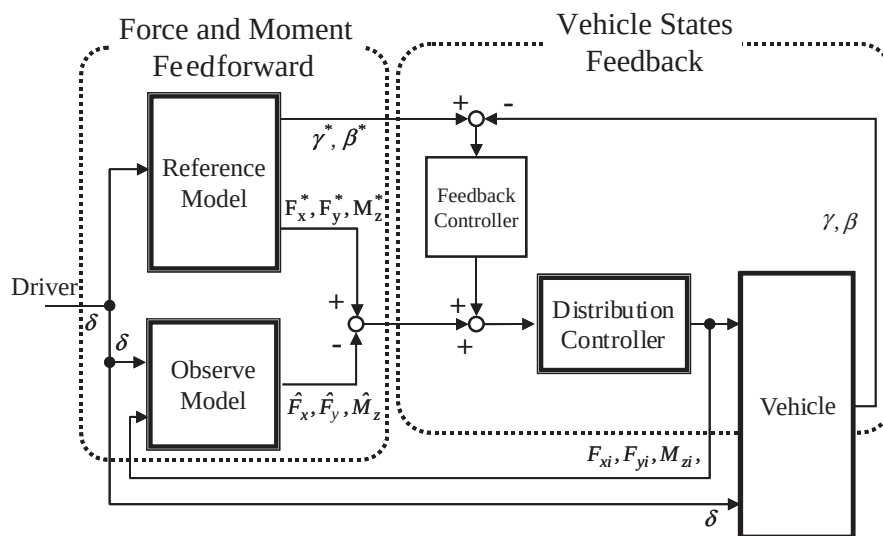
The upper layer outputs the target values to the lower layer, while the lower layer feeds back the results. The upper layer then recalculates the target value depending on the results.

This both-way communication enables each layer to cooperate with the other and maintain greater robustness corresponding to the characteristic change of the controlled system and environment.

#### 4. Adoption of force and moment control

**Figure 3** shows the proposed control flow. This flow consists of two parts: One is feedforward control of the force and moment of the vehicle and the other is feedback control of the vehicle states.

This type of system is generally called "Two-Degrees-of-Freedom Control." The main advantage of this system is that it enables the independent design of controllability and stability.<sup>8)</sup>



**Fig. 3** Vehicle dynamics control algorithm.

The feedforward control estimates the tire forces. Therefore, the part controlled as the force and moment of the vehicle, as calculated by estimating the tire force, is equal to the value obtained by the reference model.

Feedback control stabilizes the vehicle behavior as it approaches the limit region where the modeling error is large.

By combining these two parts, vehicle behavior can be made smooth and the probability of approaching the critical limit can be reduced.

## 5. Nonlinear optimum distribution method

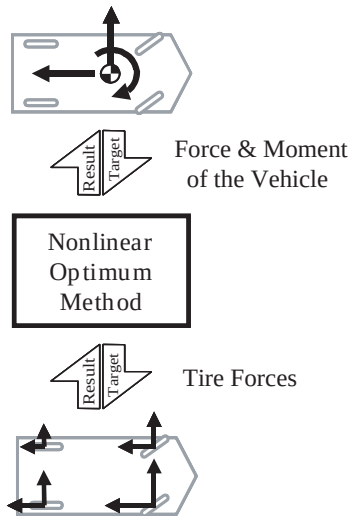
The other core technology employed by VDM is a force and moment distribution method that uses nonlinear optimization. As shown in **Fig. 4**, this method distributes the target force and moment of the vehicle to the longitudinal and lateral forces of each wheel.

It is well known that, when braking in a turn, the distribution of the braking force in proportion to the vertical load of each wheel causes the yaw moment to remain the same, if any reduction in the lateral force corresponding to an increase in the longitudinal force is ignored. The tire force, however, features non-linear saturating characteristics where the value varies with the vehicle state and road surface. Therefore, it is not easy to determine the force that should be generated at each tire to obtain the desired vehicle behavior in any driving situation.

The proposed nonlinear optimizing method is efficient in the following two points.

- When a target value that exceeds the physical limit is input, the algorithm automatically executes a trade-off based on a pre-designed performance function.
- When the target value is sufficiently small, the algorithm is able to distribute a reasonable force to each wheel, making it a redundant system regulated by the performance function.

For these reasons, a nonlinear optimum method is



**Fig. 4** Force & moment distribution control.

proposed. It is based on the following tire model.

### 5.1 Tire model

The characteristics that the tire force distribution algorithm requires of the tire model are as follows.

- Saturation of tire force
- Dependency of driving and cornering stiffness on vertical load
- Relationship between longitudinal and lateral force (friction circle)
- Fewer parameters for describing the model

The simple brush model described by Eqs. (1)-(10) satisfies these requirements.<sup>9)</sup> The characteristics of the longitudinal and lateral forces for the applied tire model are shown in **Fig. 5**.

Where  $\kappa_i$ ,  $\alpha_i$  are the slip ratio and slip angle of each tire,  $K_\alpha$ ,  $K_\kappa$  are the cornering and driving stiffness,  $K_{\alpha 0}$ ,  $K_{\kappa 0}$ , are the coefficients of stiffness corresponding to the vertical load,  $F_{zi}$  is the vertical load on each wheel, and  $\mu_i$  is the maximum tire force normalized by  $F_{zi}$ .

$$\cos \theta = \frac{\kappa_i}{\lambda} \quad \dots\dots\dots (1)$$

$$\sin \theta = \frac{K_\alpha \tan \alpha_i}{K_\kappa \lambda} \quad \dots\dots\dots (2)$$

$$K_\kappa = K_{\kappa 0} F_{zi} \quad \dots\dots\dots (3)$$

$$K_\alpha = K_{\alpha 0} F_{zi} \quad \dots\dots\dots (4)$$

$$\lambda = \sqrt{\kappa_i^2 + \frac{K_\alpha^2 \tan^2 \alpha_i}{K_\kappa^2}} \quad \dots\dots\dots (5)$$

$$\xi = 1 - \frac{K_\kappa \lambda}{3\mu_i F_{zi} (1 - \kappa_i)} \quad \dots\dots\dots (6)$$

1. Where  $\xi > 0$ ,

$$F_{xi} = -\frac{\xi^2 K_\kappa}{(1 - \kappa_i)} \kappa_i - \mu_i F_{zi} \cos \theta (1 - 3\xi^2 + 2\xi^3) \quad \dots\dots\dots (7)$$

$$F_{yi} = -\frac{\xi^2 K_\alpha \tan \alpha_i}{1 - \kappa_i} - \mu_i F_{zi} \sin \theta (1 - 3\xi^2 + 2\xi^3) \quad \dots\dots\dots (8)$$

2. Where  $\xi < 0$ ,

$$F_{xi} = -\mu_i F_{zi} \cos \theta \quad \dots\dots\dots (9)$$

$$F_{yi} = -\mu_i F_{zi} \sin \theta \quad \dots\dots\dots (10)$$

## 5.2 Nonlinear optimizing algorithm

The object is to solve for the optimum balance of the tire force. When a balance is achieved, the error between the target force and moment of the vehicle and those generated by each tire force are almost equal and, simultaneously, each ratio of generated tire force to the maximum tire force is almost equal.

The optimizing algorithm minimizes the following performance function. Sequential quadratic programming is used to solve this problem.<sup>11)</sup>

$$L = \mathbf{E}^T \mathbf{W}_E \mathbf{E} + \delta \mathbf{\kappa}^T \mathbf{W}_{\delta \mathbf{\kappa}} \delta \mathbf{\kappa} + (\mathbf{\kappa} + \delta \mathbf{\kappa})^T \mathbf{W}_{\mathbf{\kappa}} (\mathbf{\kappa} + \delta \mathbf{\kappa}) \quad \dots\dots\dots (11)$$

Where  $\mathbf{\kappa}$  is the slip ratio of each wheel.

This controller is discretized. And,  $\delta \mathbf{\kappa}$  is the operating value of this system. Next,  $\mathbf{\kappa}$  is modified by  $\delta \mathbf{\kappa}$  to  $\mathbf{\kappa} + \delta \mathbf{\kappa}$ .

This method involves calculating  $\delta \mathbf{\kappa}$  to minimize  $L$  for the linearized system around the operating point at each control sampling time.

$$\mathbf{\kappa} = [\kappa_{fl} \ \kappa_{fr} \ \kappa_{rl} \ \kappa_{rr}]^T \quad \dots\dots\dots (12)$$

$$\delta \mathbf{\kappa} = [\delta \kappa_{fl} \ \delta \kappa_{fr} \ \delta \kappa_{rl} \ \delta \kappa_{rr}]^T \quad \dots\dots\dots (13)$$

$\mathbf{E}$  is the error between the target force and the moment of the vehicle ( $F_x^*$ ,  $F_y^*$ ,  $M_z^*$ ), and that calculated from the distributed tire forces ( $\hat{F}_x$ ,  $\hat{F}_y$ ,  $\hat{M}_z$ ), and is given by Eq. (14).

$$\mathbf{E} = \begin{bmatrix} F_x^* - \hat{F}_x \\ F_y^* - \hat{F}_y \\ M_z^* - \hat{M}_z \end{bmatrix} \quad \dots\dots\dots (14)$$

The purpose of the term " $\mathbf{\kappa} + \delta \mathbf{\kappa}$ " is to equalize each ratio of tire force to the maximum force, while that of " $\delta \mathbf{\kappa}$ " is to prevent the actuator from responding too quickly.

Also,  $\mathbf{W}_E$ ,  $\mathbf{W}_{\delta \mathbf{\kappa}}$ , and  $\mathbf{W}_{\mathbf{\kappa}}$  are the weights corresponding to  $\mathbf{E}$ ,  $\delta \mathbf{\kappa}$  and  $\mathbf{\kappa}$ . The force and moment of the vehicle calculated by summing the distributed tire forces of each wheel ( $F_x$ ,  $F_y$ ,  $F_z$ ) are described by Eq. (15) using each tire force and the Jacobian ( $J$ ) value given by Eq. (16).

$$\begin{bmatrix} \hat{F}_x \\ \hat{F}_y \\ \hat{M}_z \end{bmatrix} = \sum_{i=1}^4 \begin{bmatrix} F_{xi} \\ F_{yi} \\ M_{zi} \end{bmatrix} + J \delta \mathbf{\kappa} \quad \dots\dots\dots (15)$$

$$J = \begin{bmatrix} \frac{\partial F_x}{\partial \kappa_{fl}} & \frac{\partial F_x}{\partial \kappa_{fr}} & \frac{\partial F_x}{\partial \kappa_{rl}} & \frac{\partial F_x}{\partial \kappa_{rr}} \\ \frac{\partial F_y}{\partial \kappa_{fl}} & \frac{\partial F_y}{\partial \kappa_{fr}} & \frac{\partial F_y}{\partial \kappa_{rl}} & \frac{\partial F_y}{\partial \kappa_{rr}} \\ \frac{\partial M_z}{\partial \kappa_{fl}} & \frac{\partial M_z}{\partial \kappa_{fr}} & \frac{\partial M_z}{\partial \kappa_{rl}} & \frac{\partial M_z}{\partial \kappa_{rr}} \end{bmatrix} \quad \dots\dots\dots (16)$$

Here,  $J$  is described as the function of the slip ratio ( $\kappa$ ) and the slip angle based on the tire model.

After the above preparation,  $\delta \mathbf{\kappa}$  satisfies Eq. (17), minimizing  $L$ .

$$\partial L / \partial \delta \mathbf{\kappa} = 0 \quad \dots\dots\dots (17)$$

We solve Eq. (17) for  $\delta \mathbf{\kappa}$  by substituting Eqs. (11)-(16). Finally, we obtain Eq. (18).

$$\delta \mathbf{\kappa} = (\mathbf{W}_{\delta \mathbf{\kappa}} + \mathbf{W}_{\mathbf{\kappa}} + J^T \mathbf{W}_E J)^{-1} (J^T \mathbf{W}_E \mathbf{\Delta}_E - \mathbf{W}_{\mathbf{\kappa}} \mathbf{\kappa}) \quad \dots\dots\dots (18)$$

$$\mathbf{\Delta}_E = [F_x^*, F_y^*, M_z^*]^T - \sum_{i=1}^4 [F_{xi} \ F_{yi} \ M_{zi}]^T \quad \dots\dots\dots (19)$$

## 6. Simulation

This section explains the simulations that we performed to show the effect of the proposed control method. The conventional system calculates a target yaw moment by regulating the weighted-sum of the

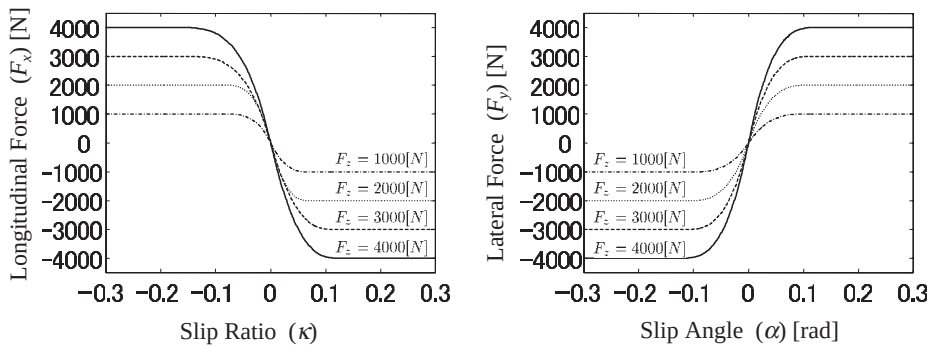


Fig. 5 Characteristics of tire force.

slip angle and the angular velocity of the vehicle. Then, if the target yaw moment is in the outward direction during cornering, it is applied by the outside front wheel. On the other hand, if the target is in the inward direction, it is applied by the three wheels other than the inside rear wheel.

### 6.1 Slalom maneuver

The control inputs and vehicle states during a slalom maneuver on a low-friction road surface are shown in **Fig. 6**.

The proposed method compensates for any imbalance in the yaw moment by means of feedforward prior to the vehicle skidding. As a result, unstable vehicle behavior is highly unlikely to occur.

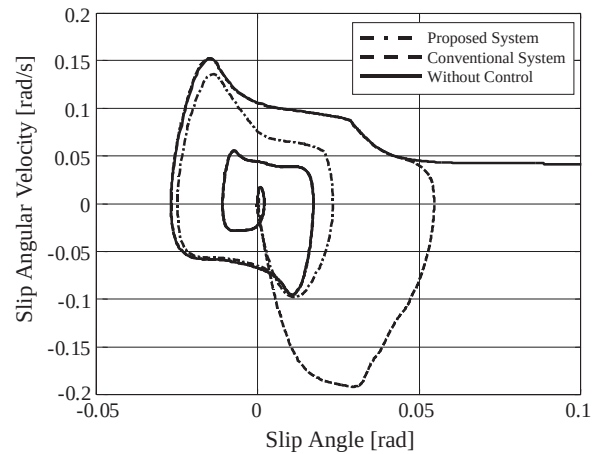
Figure 6(a) shows the phase plane trajectory of the slip angle and angular velocity of the vehicle. The proposed control draws the smallest trajectory, indicating the highest stability. Furthermore, Fig. 6(b) shows differences in the steering angle, yaw velocity and control inputs between the conventional and proposed controls. The proposed control starts applying control earlier than the conventional control. The proposed control also indicates good tracking performance between the steering angle and yaw velocity in order to start the application of control to the balance of the four wheel forces before the vehicle skids.

### 6.2 Closing the throttle during a turn maneuver

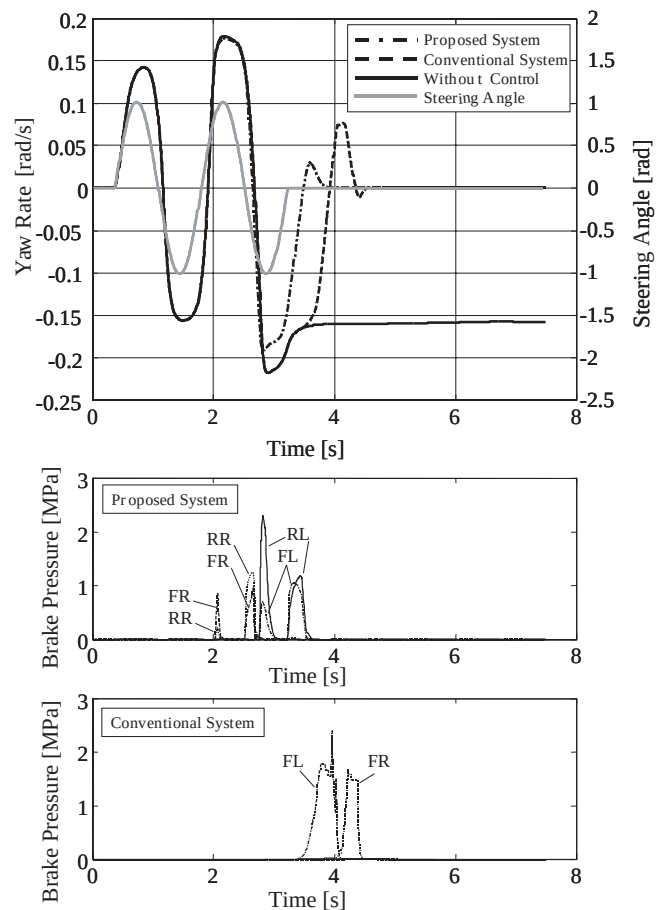
If we close the throttle during limit cornering in a rear-wheel drive vehicle, the vehicle may deviate from the desired trace line. Because deceleration slip of the rear wheels increases, the weight is transferred to the front wheels, causing the front lateral force to increase and the rear lateral force to decrease.

As shown in **Fig. 7**, variations in the slip angle and the yaw velocity increase very slowly. As a consequence, conventional control that mainly uses feedback of the vehicle states cannot produce the desired effect. This is caused by the error in the vehicle behavior being so small that it is masked by the dead zone to avoid signal noise.

On the other hand, the proposed method calculates the variance in the yaw moment caused by closing the throttle based on the vehicle model and driver inputs. This method is able to cancel the desired yaw



(a) Stability performance



(b) Steerability performance & control inputs

**Fig. 6** Slalom maneuver.



moment before the vehicle skids. Consequently, undesirable vehicle behavior almost never occurs (Fig. 8).

## 7. Conclusion

We have proposed a VDM concept that realizes

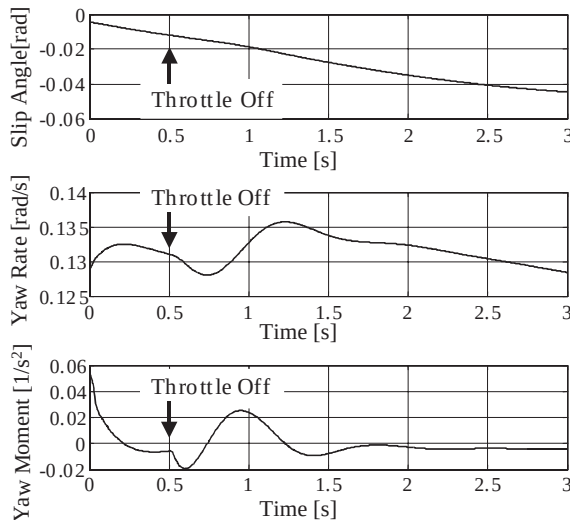


Fig. 7 Closing the throttle during a turn maneuver

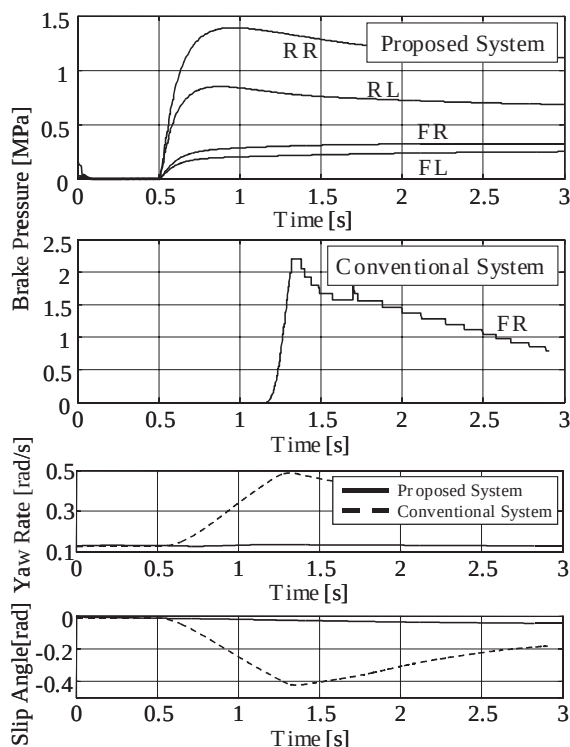


Fig. 8 Control results.

excellent steerability and stability at any time while driving. The key technologies of VDM, namely, the methods for feedforward control of the force and moment and the nonlinear optimum distribution algorithm are described. The performance of the proposed control system was confirmed by simulation.

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