

Research
Report

Development of Tire Side Force Model Based on "Magic Formula" with the Influence of Tire Surface Temperature

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タイヤ表面温度の影響を考慮したMagic Formula横力モデルの開発

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Abstract

The results of vehicle performance tests often vary depending on the season or with differences in the road surface temperature. It is believed that these changes can partially be attributed to the effect of tire surface temperature. The aim of this study is to develop a tire side force model that incorporates the influence of tire surface temperature.

The newly developed tire side force model incorporates a thermodynamic model that allows

us to consider changes in the tire surface temperature and a side force model that allows for the effects of tire surface temperature. The tire model parameters were identified using the results obtained with an indoor test facility.

The surface temperature and the side force values predicted using this model agreed well with actual measurements, proving the validity of the developed model.

Keywords

Tire model, Thermodynamics, Simulation, Measurement, Vehicle dynamics, Handling, Temperature

要 旨

季節や路面温度の違いにより車両運動性能が異なることがしばしば見られるが、一要因としてタイヤ発生力の温度依存性が考えられる。そこで、本研究では、タイヤ表面温度の影響を考慮したタイヤ横力モデルを構築することを目的とする。

このモデルは、タイヤ表面温度変化に関する熱伝熱モデルと、タイヤ表面温度の変化に伴う横

力の変化を表すモデルとで構成されている。このモデルで用いられているパラメータは、室内試験機で求めた試験結果を用いて導出される。

このモデルを用いてタイヤ試験機でのタイヤ表面温度の変化と横力の変化を予測したところ、実験結果とほぼ一致した結果が得られ、開発したモデルの妥当性を証明できた。

キーワード

タイヤモデル, 熱力学, シミュレーション, 計測法, 車両運動性能, ハンドリング, 温度

1. Introduction

In vehicle dynamics, an accurate description of the tire characteristics is extremely important to the study of a wide range of vehicle behavior. For example, in the development of control systems for improving vehicle stability and control, such as ABS and VSC (Vehicle Stability Control) systems, an accurate tire model capable of simulating a wide range of conditions is required to evaluate the systems.

In general, data obtained using indoor test facilities is used to develop highly accurate tire models. However, the use of indoor facilities introduces some problems into the measurements. One such problem is that the tire measurement input conditions do not correspond to the actual conditions under which a given tire/vehicle combination is driven.

To address this problem, the authors attempted to develop a tire model that is based on data obtained using a vehicle running on actual roads.¹⁾ At the same time, a test procedure for indoor test facilities was developed²⁾ to measure the tire characteristics under realistic driving conditions for modeling the tire force and moment properties.

Even if such modeling and measuring techniques are applied, however, other conditions such as the tire surface temperature cannot be fitted to all actual running conditions. Therefore, a new tire side force model that is capable of considering changes in the tire surface temperature was developed and is presented in this paper. The next chapter explains the basic structure of the model. Then, the deviation method for the model parameters is presented. Finally, the results calculated with the new model are compared with those obtained with the indoor test facility.

2. Mathematical model for the tire side force that considers the influence of tire surface temperature

The authors proposed two functions to incorporate into a new tire side force model that considers the dependence of tire surface temperature. These functions were:

- A thermodynamic model that describes changes

in the tire surface temperature

- A tire side force model that considers changes in the tire surface temperature

The following section explains each of these models in detail.

2.1 Thermodynamic model of the tire surface

A thermodynamic model of the tire surface was defined based on the following assumptions:

- 1) Thermal input occurs in the contact area
- 2) Thermal output also occurs in the contact area
- 3) Thermal input and output occur simultaneously in the same area

Based on these assumptions, the tire surface temperature T can be described as,

$$W \frac{dT}{dt} = q - \lambda A (T - T_0) \quad \text{***} \quad \dots \dots \dots (1)$$

where,

- W : Heat capacity
- q : Heat flux
- λ : Thermal conductivity
- A : Contact area
- T_0 : Ambient temperature

Next, the heat flux q is described assuming that the action of the tire on the road surface is changed to thermal energy, as follows.

$$q = F_y V \alpha \quad \dots \dots \dots (2)$$

where,

- F_y : Tire side force
- V : Vehicle velocity
- α : Slip angle

Then, Eq. (2) can be substituted into Eq. (1) to give,

$$\frac{dT}{dt} = \frac{1}{W} [F_y V \alpha - \lambda A (T - T_0)] \quad \dots \dots \dots (3)$$

As Eq. (3) is a differential equation that describes the relationship between the tire side force and the tire surface temperature, it can be used to calculate the surface temperature of the tire.

To solve this equation, we must derive two parameters, namely, W and λ .

2.2 Tire side force model based on "Magic Formula" and incorporating the influence of tire surface temperature

In recent years, a tire model called the "Magic

***The publisher regrets that the equation(1) was incorrect. It is reproduced correctly as of July 6, 2004.

<Error>	<Correction>
$W \frac{dT}{dt} = q - \lambda A (T - T_0)$	$W \frac{dT}{dt} = q - \lambda A (T - T_0)$

Formula" has been used for vehicle dynamics simulation. Therefore, the new tire side force model that incorporates the influence of tire surface temperature is based on the Magic Formula tire model. The equations describing the Magic Formula³⁾ tire side force model are as follows:

$$F_y(\alpha) = D_y \sin [C_y \tan^{-1} \{B_y \alpha - E_y (B_y \alpha - \tan^{-1}(B_y \alpha))\}] + S_{vy} \quad \dots\dots\dots(4)$$

$$\alpha = \alpha^* + S_{hy} \quad \dots\dots\dots(5)$$

We are required to make two assumptions regarding the influence of tire surface temperature on the side force, namely:

- 1) The side force changes in proportion to the tire surface temperature.
- 2) The coefficient of dependency of the tire surface temperature on the maximum side force has a different proportional value from that of the cornering power.

Considering the above two points, we can rewrite the Magic Formula tire side force model to include the influence of tire surface temperature, as follows:

$$F_y(\alpha, T) = D_y(T) \sin [C_y \tan^{-1} \{B_y(T) \alpha - E_y (B_y(T) \alpha - \tan^{-1}(B_y(T) \alpha))\}] + S_{vy} \quad \dots\dots\dots(6)$$

$$D_y(T) = D_{y0} \cdot \left\{ 1 + \frac{\partial \mu}{\partial T} (T - T_m) \right\} \quad \dots\dots\dots(7)$$

$$B_y(T) = \frac{K_y(T)}{C_y \cdot D_y(T)} \quad \dots\dots\dots(8)$$

$$K_y(T) = K_{y0} \cdot \left\{ 1 + \frac{\partial C_p}{\partial T} (T - T_m) \right\} \quad \dots\dots\dots(9)$$

where,

$D_y(T)$: Peak value of tire side force including the influence of the tire surface temperature

D_{y0} : Peak value of steady state tire side force model

$K_y(T)$: Cornering power (almost exactly $\partial F_y(\alpha, T) / \partial \alpha$ at $\alpha=0$) including the influence of the tire surface temperature

K_{y0} : Cornering power (almost exactly $\partial F_y(\alpha) / \partial \alpha$ at $\alpha=0$) of the steady state tire model

T_m : Average tire surface temperature during

measurement to obtain the steady state tire model data

$\partial C_p / \partial T$: Change in tire side force with tire surface temperature

$\partial \mu / \partial T$: Change in cornering power with tire surface temperature

For the above equations, two parameters are to be derived, namely, $\partial \mu / \partial T$ and $\partial C_p / \partial T$.

The next section explains the method used to derive the parameters in Eqs. (3), (7), and (9).

3. Derivation of model parameters

The measurements described below were obtained using a 205/65R15 passenger vehicle tire.

3.1 Deriving the parameters used in the function for tire surface temperature

First, let's consider the experimental methods used to derive the parameters λ and W in Eq. (3). Equation (3) can be described as,

$$\frac{dT}{dt} = \frac{F_y V \alpha}{W} - \kappa (T - T_0) \quad \dots\dots\dots(10)$$

Where κ is a new parameter, defined as,

$$\kappa = \frac{\lambda A}{W} \quad \dots\dots\dots(11)$$

Then, Eq. (10) can be described when the work performed by the tire is equal to zero, as follows.

$$\kappa = \frac{dT}{dt} \cdot \frac{1}{T - T_0} \quad \dots\dots\dots(12)$$

Using this equation, the parameter κ can be determined when the work being performed by the tire is zero and the tire surface temperature is changing. (This corresponds to the tire being at rest and cooling down.) The sequence of the slip angle change in the measurement is shown in **Fig. 1**.

The measured values of κ are shown in **Fig. 2**. The parameter κ changes with time. In this case, the parameter κ was set to the stable state average value (between 10 and 20 seconds) to remove the thermal noise from the measuring system.

Next, let's consider the experimental method used to derive parameter W . Equation (3) can be described as:

$$W = \frac{F_y V \alpha}{\frac{dT}{dt} + \kappa(T - T_0)} \quad \dots\dots\dots (13)$$

The parameter W is calculated after κ is derived. This can be measured when the work being done by the tire is other than zero. (This corresponds to the state where the tire is warming up.) So, the sequence is, for example, the period when measuring parameter κ between -22 and -2 seconds in Fig. 1.

Figure 3 shows the results calculated for parameter W . The absolute values of parameter W when the slip angles are $+10$ degrees and -10 degrees are different, and these values change with time. Then, the value of W was determined by averaging the results at slip angles of 10 degrees and -10 degrees, which were also averaged values between -15 and 2 seconds in Fig. 3, for the same reason as that for parameter κ .

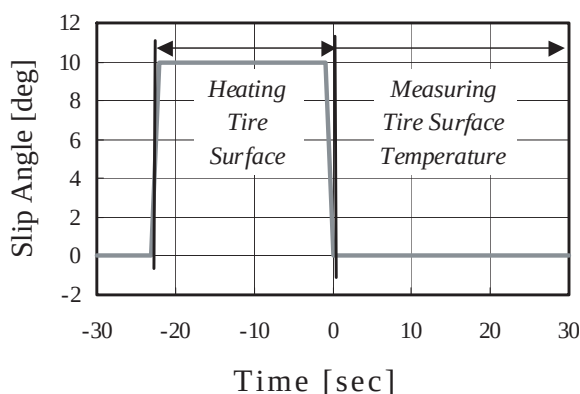


Fig. 1 Slip angle to measure the parameter κ and W .

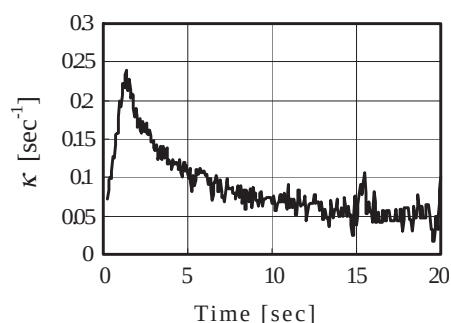


Fig. 2 Measured result of κ [Vertical load : 4600N].

3. 2 Method of deviation of parameters in Magic Formula incorporating the influence of tire surface temperature

The following describes the methods of deviation about parameters $\partial\mu/\partial T$ and $\partial C_p/\partial T$:

When Eq. (7) is differentiated by tire surface temperature T , the equation becomes:

$$\frac{d(D_y(T))/dT}{D_{y0}} = \frac{\partial\mu}{\partial T} \quad \dots\dots\dots (14)$$

In this equation, D_{y0} is the peak value of the tire side force model which uses the steady state tire measured data. Then, parameter $\partial\mu/\partial T$ is defined by the relation between the peak side force value and the tire surface temperature.

Figure 4 shows the measured data for the tire side force and tire surface temperature, as well as the results obtained for $d(D_y(T))/dT$. In this case, the slip angle is fixed to that angle that generates the

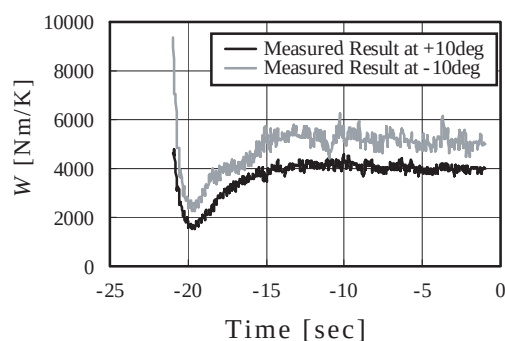


Fig. 3 Calculated result of parameter W [Vertical load : 4600N].

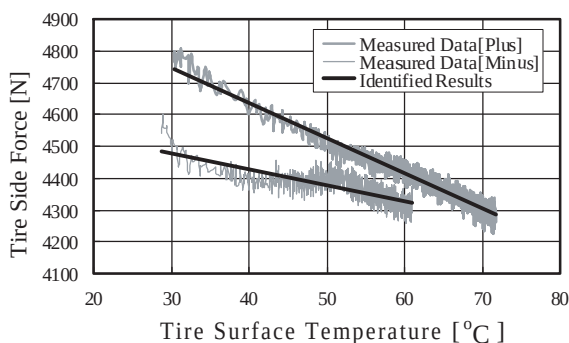


Fig.4 Measured result of peak tire side force and tire surface temperature and identified results of $d(D_y(T))/dT$ [Vertical load : 4600N].

peak side force. The gradient of the peak value of side force $d(D_y(T))/dT$ was measured at the positive and negative slip angle, and then averaged.

Finally, the following explains the method of deviation about parameter $\partial C_p/\partial T$:

When the Eq. (9) is differentiated by the tire surface temperature T , it is described as,

$$\frac{d(K_y(T))/dT}{K_{y0}} = \frac{\partial C_p}{\partial T} \dots\dots\dots(15)$$

From this equation, the parameter $\partial C_p/\partial T$ is defined as the relationship between $K_y(T)$ and the tire surface temperature.

We should note that $K_y(T)$ indicates the cornering power, as described in the previous section. In this paper, we regard the tire side force at $\alpha = 1$ [degree] or at $\alpha = -1$ [degree] to be the cornering power.

The gradient of change of the cornering power with an increase/decrease in the tire surface temperature is $d(K_y(T))/dT$. The measured results for the cornering power are shown in **Fig. 5**. The results identified for $d(K_y(T))/dT$ are also shown.

As the value of $d(K_y(T))/dT$ at 1 degree is not same as that at -1 degree, the value of $d(K_y(T))/dT$ is averaged and the parameter $\partial C_p/\partial T$ is calculated by using the average value $d(K_y(T))/dT$.

4. Comparison of newly developed tire model and test results

The simulation results of the new tire model were compared with the measured data using an indoor test facility. The test sequence is shown in **Fig. 6**.

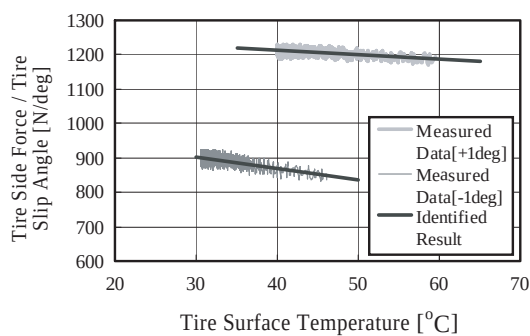


Fig. 5 Measured result of cornering and tire surface temperature and identified results of $d(K_y(T))/dT$ [Vertical load : 4600N].

The ambient temperature and the road surface temperature were maintained at the same level during measurement. To change the tire surface temperature, the angular velocity of the slip angle was changed.

For the simulation, although not mentioned above, the transient property of the side force was included by considering the first order lag denoted by σ ($= 0.6$ m), because the angular velocity of the slip angle became rather high under certain conditions, as shown in Fig. 6.

$$\sigma \frac{d\tilde{F}_y}{dt} + \tilde{F}_y = VF_y(\alpha, T) \dots\dots\dots(16)$$

The side force model under the nominal conditions was created using the sequence indicated by the dashed lines in Fig. 6.

The measured results for the tire side force and tire surface temperature versus slip angle are shown in Fig. 7, and the simulation results are as shown in Fig. 8. By comparing these figures, we can draw the following conclusion.

The tendency of the side force change agrees very well between the calculated and measured results, that is, the drop in the side force at a large slip angle, and the hysteresis of the side force as the slip angle increases and decreases in a 40-second sequence. Good agreement was also observed for the tire surface temperature.

The above proves that the newly developed model can produce results that are comparable with the test results.

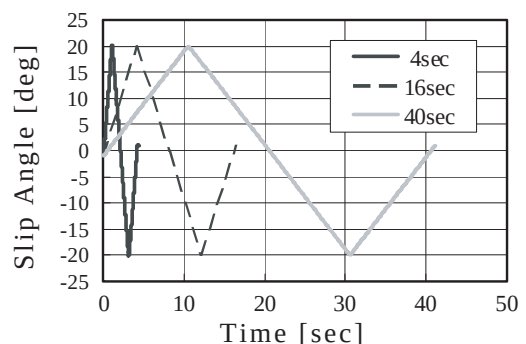


Fig. 6 Test sequence to compare the test result and simulation result.

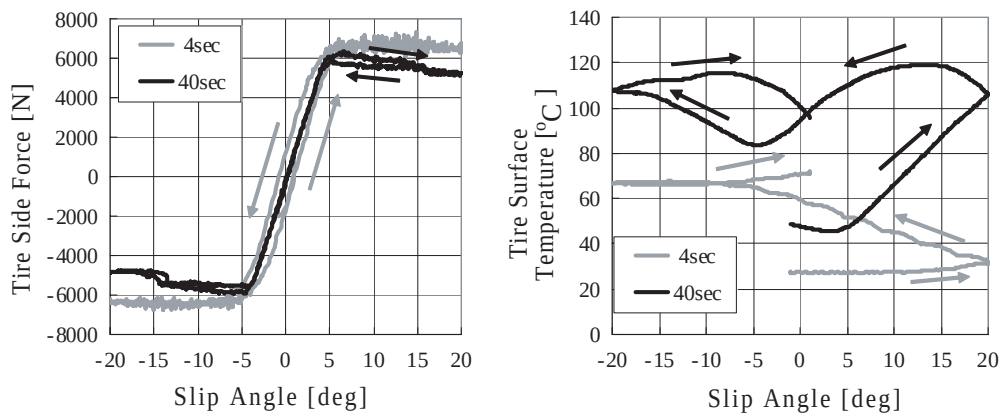


Fig. 7 The measured results of tire characteristics with different slip angular velocity.

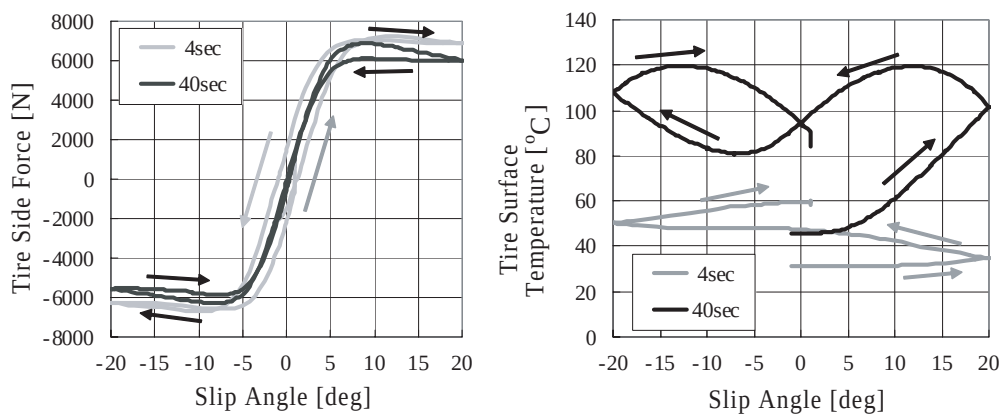


Fig. 8 The simulation results of tire characteristics with different slip angular velocity.

5. Conclusions

We can summarize the findings of this study as follows:

- (1) A new tire side force model that takes the tire surface temperature into consideration has been developed.
- (2) Methods of deriving the parameters for the new tire model were proposed and measurements were made.
- (3) The simulation results were compared with the measured results. The results obtained for the change in the tire side force and the surface temperature using the developed model agreed well with the measured values.

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