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Abstract

Although the analysis of the load distribution and slip velocity in a Metal V-belt CVT is important for the evaluation and improvement of its performance, it is difficult to develop a simulation model for analyzing power transmission by a CVT belt because of the belt's complex structure.

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> In this paper, we report the development of a simulation model that can predict fundamental characteristics of a Metal V-belt CVT with useful accuracy within a reasonable amount of calculation time. The model uses a combination

of load analysis based on equilibrium equations and slip velocity analysis based on the evaluation of the pulley displacement by FEA.

The major concerns in the calculation are the load distributions in the belt, including ring tension, element compression, and pulley clamping force, since these are the most important performance indexes in the design stage of a CVT. We confirmed the validity of the calculated results by performing a basic experimental analysis of the pulley clamping force.

Keywords CVT, Metal V-belt, Pulley, Load distribution, Deformation, Simulation

金属ベルト式CVTの性能の評価・改良には、 CVT内部で発生する荷重分布・すべり速度の解 析が重要であるが、その複雑な構造ゆえ、ベル トによる動力伝達を解析するシミュレーション モデルの作成は難しい。

本論では、金属ベルト式CVTの基本的な特性 を、実用的な精度で妥当な計算時間内に求める ことができるシミュレーションモデルの開発に

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ついて報告する。本モデルは、つり合い方程式に よる荷重の解析と有限要素解析で得たプーリ変位 によるすべり速度解析を基礎としている。

計算は、CVTの設計段階で重要な性能指標とな る、リング張力、エレメント圧縮力、プーリ挟圧 力を含むベルトの荷重分布を主対象とし、プーリ 挟圧力分布の基礎的な実験解析により、計算結果 の妥当性を確認する。

キーワード CVT, 金属ベルト, プーリ, 荷重分布, 変形, シミュレーション

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1. Introduction

Metal V-belt CVTs (Continuously Variable Transmissions) are frequently used in modern automobile transmissions. With increasing customer demand for higher quality vehicles, improvement in metal V-belt CVT performance - torque capacity, power transmitting efficiency, quietness, and so on has assumed greater importance.

Meeting customer demand requires the development of methods for analyzing metal V-belt CVT power transmission characteristics, such as changes in the distribution of load and friction force, that occur when the transmitting torque or rotational speed changes. However, because the belt has a very complex structure that includes many flexible parts and contacts, both numerical analysis and experimental analysis are quite difficult.

The purpose of this study is the development of an efficient and suitable numerical model for the evaluation of basic power transmission characteristics, including load distribution in the belt and pulley, that combines conventional load analysis using differential equations and stiffness analysis using finite element models.

2. Power transmission mechanism

The structure of a metal V-belt for automobiles has many metal elements supported by two sets of laminated rings, as shown in **Fig. 1**. Power is transmitted from the input shaft to the output shaft when the metal pulleys clamp the belt from both sides, providing torque and rotational speed to the input shaft. Behavior as a CVT is achieved by continuously varying the width of the pulley groove and changing the belt radius in each pulley.

It is well known that the metal V-belt transmits



Fig. 1 CVT belt.

most of the torque by element compression. The rest of the torque is transmitted by ring tension resulting from the friction force caused by the slip between elements and rings.^{1, 2)}

For many years, engineers have been interested in the analysis of the load distribution in the pulleys, elements, and rings, and several effective analytical methods have been developed. On the one hand, an analytical method that solves equilibrium equations was proposed.¹⁾ This method provides solutions very easily, but because it is necessary to determine the direction of the friction force between the belt and the pulley to solve the equations, it is difficult to use this method to simulate real phenomena.

On the other hand, finite element models using generic commercial FE codes^{3, 4)} and multi-body models for dynamics analysis^{5, 6)} were developed. These models seemed to be very powerful tools for the understanding of the real complex phenomena in a CVT, but the long modeling and simulation time they require make them unsuitable for the evaluation of numerous design parameters.

In the following sections, we will discuss the development of a numerical model that predicts the load distribution of a CVT within a reasonable modeling and simulation time. In the model, pulley stiffness and shaft clearance are considered in order to determine the direction of friction force between the belt and the pulley in the conventional method using equilibrium equations.

3. Calculation model

In this study, the major concerns regarding the power transmission characteristics of a metal V-belt CVT are the following load distributions:

- (1) Distribution of ring tension
- (2) Distribution of element compression
- (3) Distribution of pulley clamping force

To calculate these, we develop a simulation model with the calculation flow shown in **Fig. 2**.

First, based on the initial value of the direction of the friction force between the belt and the pulley, we calculate the load distribution in the belt (Step 1). Next, we calculate the displacement of the pulley flank on the basis of the result of Step 1 (Step 2), and determine the direction of the friction force between the belt and the pulley (Step 3). Because the calculation of the load distribution in Step 1 depends on the friction force direction obtained in Step 3, we iterate the procedure from Step 1 to Step 3 until the result in Step 3 converges to certain values.

3.1 Calculation of load distribution (Step 1)

For the calculation of load distribution, we use differential equations derived from equilibrium equations that consider the belt as a continuous body.^{1, 2)} This section describes the details.

When we consider a differential element in a belt (**Figs. 3** and **4**), the equilibrium equations are

dT - $\mu_a dF = 0$	••••••(1)
$dF - (T - C_r)d\theta = 0$	·····(2)
$dQ - 2\mu_b \cos\zeta dN + \mu_d$	$_{a}dF=0$ (3)
$dF - 2(\mu_b \sin\zeta \cos\varphi + \sin\zeta)$	$m\varphi)dN + (Q - C_e)d\theta = 0 (4)$
dFz - ($\cos \varphi$ - $\mu_b \sin \zeta$ s	$\sin \varphi dN = 0$ (5)
where.	

T: Ring tension

Q: Element compression

- dT, dQ: Increments of ring tension T and element compression Q
- dF, dN, dFz: Radial, normal, and axial forces on elements

 μ_a : Friction coefficient between element and ring

- *C_r*: Centrifugal force on rings per unit angle
- μ_b : Friction coefficient between belt (element) and pulley
- ζ : Direction of friction force between belt (element) and pulley (0, if tangential direction)
- φ : Pulley clamping angle

 C_e : Centrifugal force on elements per unit angle Considering the friction force direction, the differential equations derived from Eqs. (1) to (5) are summarized in **Table 1**. where,

nere,

- r_r : Ring radius in primary pulley
- r_e : Element radius in primary pulley

 α_1 , α_2 : Contact arc angle in each pulley

In all cases, N_1 , N_2 , and T_q depend on the values of T_a and Q_a , so we can find two equations if we provide two values from among N_1 , N_2 and T_q . Thus, we can determine the two unknown values T_a and Q_a , and obtain the ring tension T_1 , T_2 , element compression Q_1 , Q_2 , and pulley clamping force



Fig. 2 Flowchart of calculation.



Fig. 3 Equilibrium of forces that act on rings.



Fig. 4 Equilibrium of forces that act on elements.

$\frac{1}{2}$		$\frac{1}{2} = \frac{1}{2}$	
	10 ~ 1		
$T=T$ $Q=Q_1$ $\theta=0$ $\Theta=0$ $\Theta=0$ $Pri.$ S Ring tension $T=T_b$	Tent compression $T=T_a$ $T=T_a$ $T=T_2$ $T=T_2$ $T=T_2$ $T=T_2$ $T=T_2$	Element compression $T=T_a$	$T=T_2$ g tension
Ring tension:		Ring tension:	
$\frac{dT_1(\theta)}{d\theta} = -\mu_a(T_1(\theta) - C_r)$	(6)	$\frac{dT_1(\theta)}{d\theta} = \mu_a(T_1(\theta) - C_r)$	(8)
$\frac{dT_2(\theta)}{d\theta} = -\mu_a(T_2(\theta) - C_r)$	(7)	$\frac{dT_2(\theta)}{d\theta} = \mu_a(T_2(\theta) - C_r)$	(9)
Element compression:			
$\frac{dQ_1(\theta)}{d\theta} = \mu(Q_1(\theta) - C_e) + (\mu - \mu_a)(T_1(\theta) - C_r) $ (10)			
$\frac{dQ_2(\theta)}{d\theta} = \mu(Q_2(\theta) - C_e) + (\mu - \mu_a)(T_2(\theta) - C_r) $ (11)			
$(Q_1 < 0, Q_2 < 0)$			
Equivalent friction coeffic	ient:	Equivalent friction coefficient:	
$\mu_b \cos \zeta$		$\pm \mu_b \cos \zeta$	
$\mu = \frac{\mu}{\mu_b}\sin\zeta\cos\varphi + \sin\varphi$		$\mu = \frac{\mu}{\mu_b \sin \zeta \cos \varphi + \sin \varphi}$	
		(+/- for large/small torque)	
Pulley clamping force per unit angle:			
$p_1(\theta) = \mu_x(T_1(\theta) - C_r + Q_1(\theta) - C_e) $ (12)			
$p_2(\theta) = \mu_x(T_2(\theta) - C_r + Q_2(\theta) - C_e) $ (13)			
Coefficient for wedge effect of pulley clamping:			
$\mu_{e} = \frac{\cos \varphi - \mu_{b} \sin \zeta \sin \varphi}{2}$			
$\frac{\mu x}{2(\mu b \sin \zeta \cos \varphi + \sin \varphi)}$			
Pulley clamping force:			
N_1	$=\int_{0}^{\alpha_{1}}p_{1}(\theta)d\theta$	(14)	
N2	$=\int^{\alpha_2}p_2(\theta)d\theta$	(15)	
Input torque:		Input torque:	
$T_q = r_r(T_b - T_a) - r_e Q_a$	(16)	$T_q = r_r(T_b - T_a) \mp r_e Q_a$	(17)

(Index "1" means primary and "2" means secondary)

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(-/+ for large/small torque)

p_1, p_2 .

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To solve these differential equations and to

calculate the load distribution, we have to know the angle $\zeta(\theta)$ in the equations. Therefore, we estimate the angle ζ with the method described in the following section. This method is based on the idea that the angle ζ depends on the direction of the relative slip velocity between the belt and the pulley.

3.2 Calculation of pulley flank displacement (Step 2)

3.2.1 Displacement by pulley deformation

Pulleys are deformed by the pulley clamping force $p_1(\theta)$ and $p_2(\theta)$ given in Eqs. (12) and (13) in Table 1. If the amount of the displacement is much smaller than the pulley dimension, we can assume the linearity of the displacement and use the superposition principle.

Like a disk, a pulley has an axisymmetric structure, so we can define the pulley compliance, that is, the axial displacement at a point on radius rand angle position θ when a unit load is applied at a point on the same radius r and angle position 0 (see Fig. 5), as

 $k_{1f}(r, \theta)$: Fixed sheave (primary) $k_{1m}(r, \theta)$: Movable sheave (primary) $k_{2f}(r, \theta)$: Fixed sheave (secondary) $k_{2m}(r, \theta)$: Movable sheave (secondary)

The axial displacement at a point on radius r and angle position θ in the primary and secondary pulleys can then be described with the following equations, respectively:

$$D_{d1}(r_1,\theta) = \int_0^{\alpha_1} (k_{1f}(r_1,\xi-\theta) + k_{1m}(r_1,\xi-\theta)) p_1(\xi) d\xi$$
.....(18)



Fig. 5 FE model for the calculation of pulley compliance.

$$D_{d2}(r_2,\theta) = \int_{0}^{\alpha_2} (k_{2f}(r_2,\xi-\theta) + k_{2m}(r_2,\xi-\theta)) p_2(\xi) d\xi$$
.....(19)

To obtain the pulley compliance k, we first do an FE analysis as shown in Fig. 5, and obtain the axial displacement at radius r when a unit load is applied at a point on the same radius r. Next, we define an approximate function of the displacements at radius r and calculate the coefficients of the function by the least squares method. Because the displacement will be periodic about the angle θ , an appropriate periodic function can be used as the approximate function. For example:

 $k(\theta) = a\cos\theta + b\cos^2\theta + c\cos^2\theta + d\cos^2\theta + e\cos^2\theta + f$

.....(20)

Thus, if we have calculated the pulley compliances at each radius in advance, we can immediately obtain the pulley compliance at an arbitrary radius by using an interpolation.

3. 2. 2 Displacement by clearance between shaft and pulley

Because of a clearance between the shaft and the pulley, the movable sheave of a pulley can be inclined. Considering the equilibrium of moments shown in **Fig. 6**, the direction of the incline θ_0 will satisfy the following equation, which we can obtain if $p(\theta)$ is already known:

Therefore, if we neglect the contact deformation of the pulley and shaft, and define the maximum value of the incline allowed by the clearance as δ , the axial displacement at radius r and angle position θ in each pulley will be



Fig. 6 Direction of pulley incline.

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$D_{c1}(\theta) = \delta_1 r_1 \cos(\theta - \theta_{01})$	•••••(22)
$D_{c2}(\theta) = \delta_2 r_2 \cos(\theta - \theta_{02})$	•••••(23)

Finally, the axial displacement of the pulley flank by pulley deformation and incline will be

$D_1(\theta) = D_{d1}(\theta) + D_{c1}(\theta)$	·····(24)
$D_2(\theta) = D_{d2}(\theta) + D_{d2}(\theta)$	(25)

3.3 Calculation of slip velocity between belt and pulley

The axial displacement of the pulley flank causes the belt radius in the pulley to vary. The amount of the belt radius variation will be proportional to the axial displacement of pulley flank and is described as

In the following sections, we will discuss the slip velocity between the belt and pulley in the tangential direction and the radial direction separately.

3. 3. 1 Tangential direction

The velocity of a belt can be considered as a constant value v_b if we neglect the elastic deformation, and the tangential velocity of the belt v_{bc} is approximated as a value v_b when Δr is much smaller than the belt radius.

 $v_{bc}(\theta) = v_b$ (27)

On the other hand, the tangential velocity of the pulley flank depends on the belt radius, and will be the following when the angular velocity of the pulley calculated from the reduction ratio is ω :

 $v_{pc}(\theta) = (r_0 + \Delta r(\theta)) \omega$ (28) where r_0 is the reference radius of belt.

Here, we make two assumptions (see Fig. 7):

- (1) In the primary pulley, the minimum value of v_{pc} is equal to v_{bc} because the pulley drives the belt.
- (2) In the secondary pulley, the maximum value of v_{bc} is equal to v_{bc} because the belt drives the pulley.

With these assumptions, we can determine the reference radius of belt r_0 in each pulley, and can therefore calculate the relative velocity between the pulley and the belt in the tangential direction as follows:

In addition, the slip velocity caused by the play of elements²⁾ will be added to the relative velocity in the primary pulley.

3.3.2 Radial direction

In the radial direction, if the pulley deformation is sufficiently small, the radial velocity of the pulley flank can be neglected, and only the variation of Δr will cause the relative velocity between the belt and the pulley:

$$v_{rr}(\theta) = \omega \frac{d}{d\theta} \Delta r(\theta)$$
(30)

Therefore, if we assume that the direction of the relative velocity between the belt and the pulley is equal to the direction of the friction force between them, we can obtain ζ , which is used in the calculation of the load distribution:

In the actual calculation, we start the calculation with initial values of $\zeta(\theta)$, and iterate the calculation of the load distribution until the calculated result of $\zeta(\theta)$ converges to certain values.

4. Calculation result

The simulation model described in the previous section was used to perform a calculation for a CVT for passenger cars. **Figures 8**, **9**, **10** and **11** show the results at a reduction ratio of 1. For the comparison,



Fig. 7 Relative velocity between belt and pulley.

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the results for a rigid pulley with zero clearance are also shown in the same figures.

From these results, it is clear that deformation and incline of the pulley have a small effect on ring tension and element compression, but have a large effect on the pulley clamping force.

For the experimental evaluation of the influence of pulley deformation on the pulley clamping force, we measured the strain of an element in the clamping force direction. Because of the difficulty of the measurement for actual pulleys for which the reduction ratio can change, we used pulleys at a fixed reduction ratio (see **Fig. 12**). We prepared two kinds of pulleys (that is, solid and coupled pulleys for the emulation of rigid and flexible pulleys) to observe the qualitative change caused by the difference in pulley stiffness.

In Figs. 11 and 12, we can observe the common features in the measured and calculated results when pulley stiffness is considered. For example, there are the changes of the peak level and peak position at the outlet side of each pulley when torque increases. From a comparison of Figs. 11 and 12, it is clear that the calculated result corresponds to the measured result qualitatively, and it can be considered that calculation using this model is reasonable.

Though more quantitative and detailed measurement is needed to validate calculation results in the future, it is important that we are able to obtain a reasonable solution with this model. Furthermore, on a personal computer, the required CPU time for the calculation of one condition with this model is 100 - 500 seconds, much shorter than the required CPU time for analysis using a



Fig. 8 Calculation result: ring tension.



Fig. 9 Calculation result: element compression.



Fig. 10 Calculation result: direction of friction force between belt and pulley.



Fig. 11 Calculation result: pulley clamping force.



Fig. 12 Measurement result: element strain in pulley clamping direction.

typical FE model (several days). This model should therefore be especially effective for the calculation of many conditions in the design stage.

5. Conclusion

The following summarizes the main results obtained in this study:

(1) With the use of conventional differential equations based on equilibrium equations and pulley compliance based on FE analysis, an effective calculation model of the load distribution in CVT has been developed.

(2) Pulley deformation greatly influences the calculation result of the load distribution, and can be considered an important factor in achieving a reasonable calculation.

(3) The calculated result of pulley clamping force corresponds qualitatively to the measured result.

(4) This model can be considered a base for evaluating CVT performance, including belt strength and power transmitting efficiency.

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