

Abstract

To improve the performance of vehicle dynamics control systems, it is important to be able to estimate the friction force characteristics between the tires and the road. In this paper, we estimate the radius of the tire friction circle by using the relationship between the Self-Aligning Torque (SAT), and the lateral and longitudinal forces acting on each tire. Then, we propose a vehicle dynamics control system for four-wheel distributed traction/braking and four-wheel distributed steering that is based on an on-line nonlinear optimization algorithm that minimizes the maximum μ rate of the four tires by using the estimation of the radius of the tire friction circle.

Keywords

Vehicle dynamics control, Self aligning torque, Vehicle dynamics integrated management, Tire friction circle

車両運動制御における運動性能を向上させるためには、タイヤと路面の間の摩擦特性を推定する ことが重要である。この報告では、セルフアライ ニングトルク(SAT) やタイヤ発生力を利用して各

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輪の摩擦円を推定する。さらに推定された各輪の 摩擦円を利用し、4輪のµ利用率最大値を最小に するオンライン非線形最適化アルゴリズムに基づ く4輪独立操舵・制駆動統合制御則を導出した。

^{キーワード} 車両運動制御, セルフアライニングトルク, 車両運動総合マネジメント, タイヤ摩擦円

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1. Introduction

In turns with low lateral accelerations, the characteristics of Self-Aligning Torque (SAT) become more saturated than those of the lateral force, such that an algorithm for estimating the tire characteristics based on the relationship between SAT and the lateral force has been developed.^{1, 2)}

While such methods can be applied to pure lateral slip motion, we clarified the relationship between the tire grip margin and SAT with combined lateral and longitudinal slip by using a brush model^{3, 4)} and are proposing a method for estimating the radius of the tire friction circle with combined lateral and longitudinal slip.⁵⁾ Furthermore, we are proposing a vehicle dynamics control system for four-wheel distributed steering and four-wheel distributed traction/braking systems, that aims to optimize the tire friction circle of each wheel. Then, the effects of the vehicle dynamics control system on four-wheel distributed steering and four-wheel distributed traction/braking systems are shown by simulations.

2. Estimation of tire friction circle

Each tire grip margin is estimated from the SAT, longitudinal and lateral force of each individual tire.⁵⁾ The tire grip margin ε , normal SAT T_{SAT0} and SAT model rate γ_{SAT} are defined as

$$\varepsilon \equiv 1 - \frac{\sqrt{F_x^2 + F_y^2}}{F}, \qquad (1)$$

$$T_{SAT0} \equiv \left\{ \frac{l}{6} + \frac{2l}{3} \frac{F_x}{K_\beta} \right\} F_y, \qquad (2)$$

$$\gamma \equiv \frac{T_{SAT}}{T_{SAT0}}, \qquad (3)$$

where, l: ground contact length, K_{β} : cornering stiffness, F: radius of tire friction circle, F_x , F_y : longitudinal and lateral force, and T_{SAT} : self-aligning torque (SAT). By using the brush model, the relationship between the SAT model rate, longitudinal force, and tire grip margin can be derived as

$$\left(\frac{1}{6} + \frac{2}{3}\frac{F_x}{K_\beta}\right)\gamma_{SAT} \left(1 + \varepsilon^{1/3} + \varepsilon^{2/3}\right)^2$$

$$=\frac{1}{2}\varepsilon\left(1+\varepsilon^{1/3}+\varepsilon^{2/3}\right)+\frac{3}{5}\cdot\frac{F_x}{K_{\beta}}\cdot\left(1+2\varepsilon^{1/3}+3\varepsilon^{2/3}+4\varepsilon\right).$$
....(4)

Figure 1 shows the tire grip margin ε as a function of the SAT model rate γ_{SAT} and normalized longitudinal force F_x/K_β , as calculated from Eq. (4). This figure shows that the tire grip margin can be described by a monotonous function of the SAT model rate and the normalized longitudinal force. Then, the tire grip margin can be estimated from γ_{SAT} and F_x/K_β by using a three-dimensional map as shown in Fig. 1.

Furthermore, the radius of the tire friction circle F_0 can be estimated as:

3. Vehicle dynamics control for four-wheel distributed steering and traction/braking system

3.1 Hierarchical control structure

The hierarchical control structure shown in **Fig. 2** is adopted for vehicle dynamics control.⁶⁾

[Vehicle Dynamics Control]

This layer calculates the target force and the moment of the vehicle in order to achieve the desired vehicle motion corresponding to the driver's pedal input and the steering wheel angle. The designed target force and moment also satisfy the robust stability condition.⁷⁾

[Force & Moment Distribution]

The target force and moment of the vehicle motion are distributed to the target forces of each wheel, based on the tire grip margin in this layer.



Fig. 1 Relationship between γ , F_x/K_β and e.

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[Wheel Control]

This layer controls the motion of each wheel so as to achieve the target force.

This paper describes a force & moment distribution algorithm in detail.

3.2 Nonlinear optimal control

A vehicle model is described using the coordinates shown in **Fig. 3**, with the X-axis being the direction of the target resultant force and the Y-axis being the direction perpendicular to the X-axis. The μ rate γ_i (*i* = 1, 2, 3, 4) of each wheel is defined as

where, F_{xi} , F_{yi} : longitudinal and lateral force of the i-th wheel, F_i : the radius of the friction circle of the i-th wheel. In this study, the μ rate of each wheel is



Fig. 2 Hierarchical vehicle dynamics management algorithm.



Fig. 3 Coordinate system corresponding to resultant force.

controlled so as to have the same value as

 $\gamma_i = \gamma_i = \gamma_i$, (*i*, *j* = 1, 2, 3, 4)(7) Furthermore, the angle between the direction of the friction force and the X-axis is expressed as q_i (*i* = 1, 2, 3, 4), as shown in **Fig. 4**. In the second layer of hierarchical control, the value of q_i that minimizes γ by satisfying the following constraints is calculated by using Sequential Quadratic Programming (SQP).

$$\gamma \sum_{i=1}^{4} F_i \cos q_i = F_{x0} \dots (8)$$

$$\gamma \sum_{i=1}^{4} F_i \sin q_i = F_{y0} \dots (9)$$

$$\gamma \sum_{i=1}^{4} F_i \left(-d_i \cos q_i + l_i \sin q_i \right) = M_{z0} \dots (10)$$

Eq. (8) represents a constraint indicating that the resultant force in the X-axis direction is the target value F_{x0} . Eq. (9) represents a constraint indicating that the resultant force in the Y-axis direction is target value F_{y0} . Eq. (10) represents a constraint indicating that the moment around the center of gravity of the vehicle is the desired yaw moment M_0 . By eliminating γ from Eqs. (8)-(10), the constraints of q_i can be obtained.

$$\sum_{i=1}^{4} F_i \left\{ \left(-F_{x0} d_i - M_{z0} \right) \cos q_i + F_{x0} l_i \sin q_i \right\} = 0 \dots \dots (11)$$

$$\sum_{i=1}^{4} F_i \left\{ -F_{y0} d_i \cos q_i + \left(F_{y0} l_i - M_{z0} \right) \sin q_i \right\} = 0 \dots \dots (12)$$

The performance function J that minimizes γ is defined as

where, d_{0,l_0} : constants of a typical moment arm. In this paper, we use the following constants.



Fig. 4 Relationship between direction of target resultant force and direction of friction force.

Since the numerator on the right side of Eq. (13) is a constant, maximizing *J* implies minimizing γ . By substituting Eqs. (8)-(10) into Eq. (13), *J* can be rewritten as,

$$J = d_0^2 F_{x0} \sum_{i=1}^{4} F_i \cos q_i + l_0^2 F_{y0} \sum_{i=1}^{4} F_i \sin q_i$$

$$+ M_{z0} \sum_{i=1}^{4} F_i \left(-d_i \cos q_i + l_i \sin q_i \right)$$

$$= \sum_{i=1}^{4} F_i \left\{ \left(d_0^2 F_{x0} - d_i M_{z0} \right) \cos q_i + \left(l_0^2 F_{y0} + l_i M_{z0} \right) \sin q_i \right\}$$

.....(16)

Then, the optimization problem can be formulated as follows.

Problem 1:

We need to find q_i (i = 1, 2, 3, 4), that maximizes the performance function (16) with constraint equations (11) and (12).

Problem 1 can be solved by using SQP. First, $\sin q_i$ and $\cos q_i$ can be linearized around q_{i0} as:

 $\sin q_i = \sin q_{i0} + \cos q_{i0} (q_i - q_{i0}) \cdots (17)$

 $\cos q_i = \cos q_{i0} + \sin q_{i0} (q_i - q_{i0})$(18) Then, constraint equations (11) and (12) can be expressed as follows.

$$\sum_{i=1}^{4} F_i \left\{ \left(F_{x0} d_i + M_{z0} \right) \sin q_{i0} + F_{x0} l_i \cos q_{i0} \right\} q_i$$

=
$$\sum_{i=1}^{4} F_i \left\{ \left(F_{x0} d_i + M_{z0} \right) \left(q_{i0} \sin q_{i0} + \cos q_{i0} \right) + F_{x0} l_i \left(q_{i0} \cos q_{i0} - \sin q_{i0} \right) \right\} \dots (19)$$

$$\sum_{i=1}^{4} F_i \left\{ F_{y0} d_i \sin q_{i0} + \left(F_{y0} l_i - M_{z0} \right) \cos q_{i0} \right\} q_i$$

=
$$\sum_{i=1}^{4} F_i \left\{ F_{y0} d_i \left(q_{i0} \sin q_{i0} + \cos q_{i0} \right) \right\}$$

$$+ (F_{y0}l_i - M_{z0})(q_{i0}\cos q_{i0} - \sin q_{i0}) \} \cdots \cdots (20)$$

Next, $\sin q_i$ and $\cos q_i$ can be approximated around q_{i0} by a 2nd order Taylor expansion as:

Then, performance function (16) can be expressed as

$$J = \sum_{i=1}^{4} F_i \left[-\frac{1}{2} \left\{ \left(d_0^2 F_{x0} - d_i M_{z0} \right) \cos q_{i0} + \left(l_0^2 F_{y0} + l_i M_{z0} \right) \sin q_{i0} \right\} q_i^2 + \left\{ \left(d_0^2 F_{x0} - d_i M_{z0} \right) \left(q_{i0} \cos q_{i0} - \sin q_{i0} \right) \right\} \right\}$$

where,

$$X_{Ni} = (d_0^2 F_{x0} - d_i M_{z0})(q_{i0} \cos q_{i0} - \sin q_{i0}) + (l_0^2 F_{y0} + l_i M_{z0})(q_{i0} \sin q_{i0} + \cos q_{i0}), \dots (25)$$

$$X_{Di} = \left(d_0^2 F_{x0} - d_i M_{z0}\right) \cos q_{i0} + \left(l_0^2 F_{y0} + l_i M_{z0}\right) \sin q_{i0} ,$$

Assuming that $X_{Di} > 0$, the problem of maximizing J can be rewritten as the problem minimizing

by the variable transformation

$$p_i = \sqrt{F_i X_{Di}} \left(q_i - X_i \right).$$
(29)

Further, linearized constraint equations (19) and (20) can be expressed as

where,

$$A_{\rm li} = \sqrt{\frac{F_i}{X_{Di}}} \cdot \left\{ \left(F_{x0} d_i + M_{z0} \right) \sin q_{i0} + F_{x0} l_i \cos q_{i0} \right\}, \dots (31)$$

$$A_{2i} = \sqrt{\frac{F_i}{X_{Di}}} \cdot \left\{ F_{y0} d_i \sin q_{i0} + \left(F_{y0} l_i - M_{z0} \right) \cos q_{i0} \right\}, \dots (32)$$
$$B_1 = \sum_{i=1}^{4} F_i \left[\left(F_{x0} d_i + M_{z0} \right) \left\{ \left(q_{i0} - X_i \right) \sin q_{i0} + \cos q_{i0} \right\} + F_{x0} l_i \left\{ \left(q_{i0} - X_i \right) \cos q_{i0} - \sin q_{i0} \right\} \right], \dots (33)$$

$$B_{2} = \sum_{i=1}^{4} F_{i} \Big[F_{y0} d_{i} \Big\{ \big(q_{i0} - X_{i} \big) \sin q_{i0} + \cos q_{i0} \Big\} \\ + \big(F_{y0} l_{i} - M_{z0} \big) \Big\{ \big(q_{i0} - X_{i} \big) \cos q_{i0} - \sin q_{i0} \Big\} \Big] \dots (34)$$

Since minimizing *K* implies minimizing the Euclidian norm of $\boldsymbol{p} = [p_1 \ p_2 \ p_3 \ p_4]^T$, the optimal solution can be derived by using pseudo-inverse matrix calculation as

where, A^+ represents the pseudo-inverse of matrix A. Then,

$$\boldsymbol{q} = diag \begin{bmatrix} \frac{1}{\sqrt{F_{1}X_{D1}}} & \frac{1}{\sqrt{F_{2}X_{D2}}} & \frac{1}{\sqrt{F_{3}X_{D3}}} & \frac{1}{\sqrt{F_{4}X_{D4}}} \end{bmatrix}$$
$$\cdot \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \end{bmatrix}^{+} \cdot \begin{bmatrix} B_{1} \\ B_{2} \end{bmatrix} + \begin{bmatrix} X_{1} \\ X_{2} \\ X_{3} \\ X_{4} \end{bmatrix}, \dots \dots \dots (36)$$

where,

 $\boldsymbol{q} = \left[q_1 \ q_2 \ q_3 \ q_4 \right]^T.$

Then, recursion formula solving of optimal q_i based on SQP is shown as follows.

<u>STEP 1</u> To satisfy the condition $X_{Di} > 0$, the initial value of q_{i0} is calculated as

$$q_{i0} = \tan^{-1} \frac{l_0^2 F_{y0} + l_i M_{z0}}{d_0^2 F_{x0} - d_i M_{z0}}.$$
 (37)

By substituting Eq. (37), X_{Di} can be rewritten as

$$X_{Di} = \left(d_0^2 F_{x0} - d_i M_{z0}\right) \cos q_{i0} + \left(l_0^2 F_{y0} + l_i M_{z0}\right) \sin q_{i0}$$
$$= \sqrt{\left(d_0^2 F_{x0} - d_i M_{z0}\right)^2 + \left(l_0^2 F_{y0} + l_i M_{z0}\right)^2}$$
$$\cdot \cos\left(q_{i0} - \tan^{-1} \frac{l_0^2 F_{y0} + l_i M_{z0}}{d_0^2 F_{x0} - d_i M_{z0}}\right)$$
$$= \sqrt{\left(d_0^2 F_{x0} - d_i M_{z0}\right)^2 + \left(l_0^2 F_{y0} + l_i M_{z0}\right)^2} > 0 \cdot \cdot (38)$$

<u>STEP 2</u> q_i is calculated from Eq. (36).

<u>STEP 3</u> We define a penalty function as

where,

$$J_{Fy}(\boldsymbol{q}) = \sum_{i=1}^{4} F_i \left\{ -F_{y0} d_i \cos q_i + \left(F_{y0} l_i - M_{z0} \right) \sin q_i \right\}.$$
(41)

 $J_{Fx}(q)$ corresponds to the left side of Eq. (11), and $J_{Fy}(q)$ corresponds to the left side of Eq. (12). If $P(q) < P(q_0)$, then

 $q_{0=}q$,(42) and we can go to STEP 2. Otherwise, \hat{q} which satisfies $P(\hat{q}) < P(q_0)$ is a straight line searched in $[q, q_0]$. Then,

 $q_0 = \hat{q}$,....(43) and we can go to STEP 2.

We know that SQP is one of the most effective methods of nonlinear optimization.⁸⁾ Further, optimal γ can be obtained as

$$\gamma = \frac{\left(d_0 F_{x0}\right)^2 + \left(l_0 F_{y0}\right)^2 + M_{z0}^2}{\sum_{i=1}^4 F_i \left\{ \left(d_0^2 F_{x0} - d_i M_{z0}\right) \cos q_i + \left(l_0^2 F_{y0} + l_i M_{z0}\right) \sin q_i \right\}}$$
(44)

The magnitude and direction of each tire force are translated to the steer angle and longitudinal force of each wheel, and these are controlled in the last layer of the hierarchical control. The lateral and longitudinal forces can be obtained as

Further, by using a brush model, the slip angle of each wheel can be calculated as

where,

 K_s : braking stiffness, K_{α} : cornering stiffness.

4. Simulation results

Figure 5 shows the simulation results for fourwheel distributed steering and traction/braking control in the case of a single-shot sine-wave steering maneuver at v = 22 m/s. A linear model of the vehicle is used for the reference of control. The μ rate is controlled such that it is the same for each wheel, and is restricted to a maximum value of 0.95, so that the grip margin is uniform, while the limit on the friction circles is satisfied. This implies that the 特

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tire forces are used efficiently in the friction circles. **Figure 6** compares the maximum resultant force of the simulation result for a single-shot sine-wave



Fig. 5 Simulation result for four-wheel distributed steering and traction/braking control (single-shot sine-wave steering at v = 22 m/s).



Fig. 6 Comparison of maximum resultant force of simulation result for a single-shot sine-wave steering maneuver at v = 22 m/s.

steering maneuver at v = 22 m/s. This figure shows that the limited performance (maximum resultant force) is improved through the use of integrated control. The maximum resultant force is improved by 7.8 % as a result of adding active front-rear steering control to four-wheel distributed traction/braking control, and by a further 2.5 % as a result of adding an independent steering mechanism (four-wheel distributed steering).

5. Conclusion

In this paper, the radius of each tire friction circle is estimated from the relationship between the Self-Aligning Torque (SAT) and the lateral and longitudinal forces acting on each tire, and a method of vehicle dynamics control that uses the tire friction circle of each wheel for the optimum is proposed. A distribution algorithm using Sequential Quadratic Programming (SQP) is used to calculate the magnitude and direction of the tire forces, such that they satisfy the constraints corresponding to the target force and moment of the vehicle motion and minimize each tire μ rate. The proposed algorithm, being characterized by pseudo-inverse matrix calculation, features high-speed calculation and accurate calculation that satisfy the constraints, so that it can be applied to four-wheel distributed steering and four-wheel distributed traction/braking systems for which the optimization of the eight parameters is necessary. Then, the effectiveness of the vehicle dynamics control for four-wheel distributed steering and four-wheel distributed traction/braking systems is shown by simulations.

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