



Brief Report

# Optimum Design for Fluid and Fluid-thermal Problems in Low Reynolds Number Flow by Density Based Topology Optimization

Tadayoshi Matsumori, Tsuguo Kondoh, Atsushi Kawamoto and Tsuyoshi Nomura

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## 1. Introduction

Topology optimization<sup>(1)</sup> was first applied to fluid problems by Borrvall and Petersson<sup>(2)</sup> in 2003. For expressing a solid object in fluid during the optimization process, they introduced a body force expression using a design variable based on the Brinkman penalization method and applied the method to the Stokes flow problem. The design variable that represents the solid and fluid states is the same as that in a density based topology optimization. This method has been extended to not only the fluid problem governed by the Navier-Stokes equation but also the advection-diffusion equation.<sup>(3,4)</sup>

This report deals with density based topology optimization for fluid and fluid-thermal problems. For fluid problems, we discuss the proposed parameter settings for stabilizing the optimization processes based on Borrvall's method and objective functions for minimizing the drag force and maximizing the lift force.<sup>(5)</sup> For fluid-thermal problems, we particularly focus on designing heat exchangers and present a design method for maximizing the heat transfer of a heat exchanger under constant input power.<sup>(6)</sup>

## 2. Problem Setting for Topology Optimization

### 2.1 Design Variable for Expressing Solid in Fluid

Following Borrvall's paper,<sup>(2)</sup> the solid domain in fluid flow is modeled as an idealized porous media using a fluid fraction,  $\gamma$ , which varies continuously from zero to one:  $\gamma = 0$  for solid and  $\gamma = 1$  for fluid. For topology optimization,  $\gamma$  is utilized as a design variable. Note that this setting is essentially the same as the density based method in structural optimization.

### 2.2 Flow Field Modeling

The governing equations of an incompressible steady flow, i.e., the equation of continuity and the Navier-Stokes equation, are given for the normalized velocity,  $\mathbf{u}$ , and the normalized pressure,  $p$ , as

$$\nabla \cdot \mathbf{u} = 0, (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u} + \mathbf{F}. \quad (1)$$

$\text{Re} \equiv \rho UL/\mu$  is the Reynolds number defined by the characteristic velocity  $U$ , the characteristic length  $L$ , the fluid density  $\rho$ , and the viscosity  $\mu$ . For topology optimization, the body force  $\mathbf{F}$  is modeled as  $\mathbf{F} = -\alpha(\gamma)\mathbf{u}$ , assuming a resistive force due to the porous media in the flow, and  $\alpha(\gamma)$  is expressed as  $\alpha(\gamma) = \alpha_{\max} q(1 - \gamma)/(q + \gamma)$ . Here,  $q$  is a positive parameter used for tuning the function profile of  $\alpha(\gamma)$ , and  $\alpha_{\max}$  should always be large enough regardless of  $\text{Re}$  so that the solution of Eq. (1) at the solid domain ( $\gamma = 0$ ) substantially yields  $\mathbf{u} = \mathbf{0}$ . Hence,  $\alpha_{\max}$  is defined, depending on the  $\text{Re}$ :  $\alpha_{\max} = (1 + 1/\text{Re})\chi$ .<sup>(5)</sup>  $\chi$  is a positive value that is much larger than 1. In the current study, parameters  $q$  and  $\chi$  are set to  $10^{-2}$  and  $10^4$ , respectively.

### 2.3 Thermal Field Modeling

In the fluid-thermal problem, the governing equations in the fluid and solid domains are different. As the distribution of the fluid and the solid domains can only be determined after the optimization, one cannot know a priori which equation should be adopted at each point. A simple way to treat this issue is to unify the two equations into one equation by using  $\gamma$ . The normalized governing equation becomes

$$\gamma(\mathbf{u} \cdot \nabla) T = \frac{1}{\text{Re Pr}} \{ (1 - \gamma)K + \gamma \} \nabla^2 T + \frac{Q}{\text{Re Pr}}, \quad (2)$$

where  $\text{Pr}$  is the Prandtl number and  $K \equiv k_s/k_f$  is the ratio of the solid and fluid thermal conductivities,  $k_s$  and  $k_f$ .

$Q$  is the heat generation per unit time and volume. In this way, the heat transfer problem can be solved for both the fluid domain and the solid domain with only a single Eq. (2) throughout the optimization process.

### 3. Numerical Examples

#### 3.1 Drag Minimization and Lift Maximization

Figure 1 shows a problem setting where the geometry of the computational domain and the related boundary conditions are depicted. Optimization problems for minimizing the drag force,  $D$ , under a constant solid volume,  $V_0$ , or maximizing the lift force,  $L$ , under a constrained drag force,  $D_0$ , are formulated as follows:

$$\begin{aligned} &\text{Minimize } D, \\ &\quad \gamma \in [0,1] \\ &\text{subject to } \int_{\Omega_D} (1-\gamma) d\Omega \geq V_0. \end{aligned} \tag{3}$$

$$\begin{aligned} &\text{Maximize } L, \\ &\quad \gamma \in [0,1] \\ &\text{subject to } \int_{\Omega_D} (1-\gamma) d\Omega \geq V_0, D \leq D_0. \end{aligned} \tag{4}$$

In our formulation using  $\gamma$ , the domain integration of  $-F$  expresses the drag and lift forces due to the action-reaction law:

$$D = \int_{\Omega_D} \alpha u d\Omega, L = \int_{\Omega_D} \alpha v d\Omega, \tag{5}$$

where  $u$  and  $v$  are the  $x$ - and  $y$ -components of  $\mathbf{u}$ , respectively. Other expressions of drag force and lift force are shown in Ref. (5).

Figures 2 and 3 show the optimized results based on Eqs. (3) and (4) using Eq. (5). These problems are solved by COMSOL Multiphysics (COMSOL AB) and a gradient based optimization solver, SNOPT.<sup>(7)</sup> In the drag minimization problem, we can successfully obtain minimum drag shapes up to  $Re = 2000$ . The optimized body shape of maximizing lift force inclines against the flow direction and has a trailing edge shape that suggests the usefulness of the flap-like structure.

#### 3.2 Maximization of Heat Transfer

Figure 4 shows a problem setting for maximizing heat transfer under constant input power,  $P$ . To keep  $P$  constant, we introduce the following integral equation with respect to the inlet pressure,  $p_{in}$ <sup>(6)</sup>:

$$\int_{\Gamma_{in}} p_{in} u d\Gamma = P. \tag{6}$$

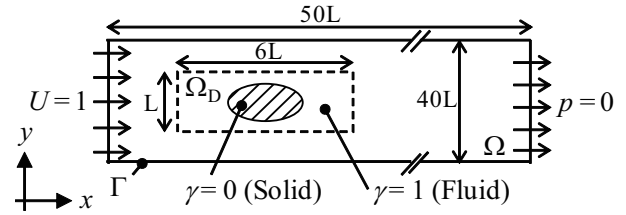


Fig. 1 Problem setting for flow optimization.

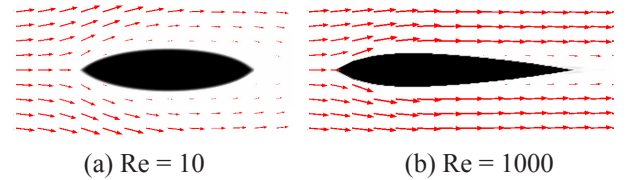


Fig. 2 Optimized results in drag minimization.



Fig. 3 Optimized results in lift maximization for  $Re = 100$  with constrained drag force.

The system is augmented by including  $p_{in}$  as a variable, where  $p_{in}$  specifies the inlet boundary condition. Consequently, for the flow field computation we obtain the system of integro-differential equations. Note that, in this case,  $Re$  is redefined based on  $P$ :  $P = (\mu^3/\rho^2 L) Re^3$ .

When a temperature-independent heat source  $Q$  is defined in the solid domain, e.g.,  $Q = (1-\gamma)/V_s$ , where  $V_s$  is a solid area, we formulate an optimization problem for maximizing heat transfer under a constant solid volume,  $V_0$ , as follows:

$$\begin{aligned} &\text{Minimize } \int_{\Omega_D} (1-\gamma) T d\Omega, \\ &\quad \gamma \in [0,1] \\ &\text{subject to } V_s \geq V_0. \end{aligned} \tag{7}$$

Figure 5 shows the optimized results with  $Re = 10, 100, K = 1, V_0 = 0.5V$ , where  $V$  is the entire design

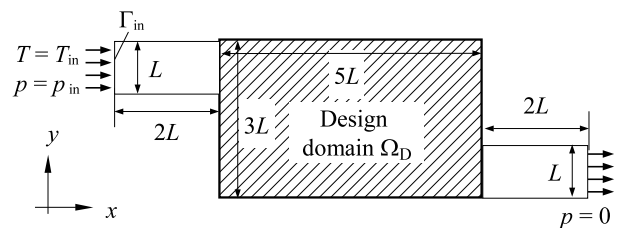
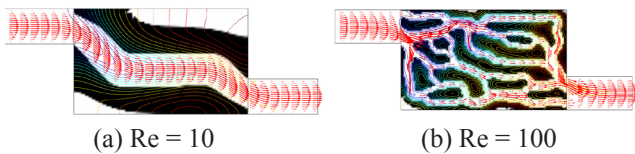


Fig. 4 Problem setting for heat transfer optimization.



**Fig. 5** Optimized results of Eq. (7) with  $K = 1$ ,  $V_0 = 0.5V$ : Contour lines represent temperature, and the vector plot shows velocity.

domain area.  $Re$  proportional to  $\sqrt[3]{P}$  is the main factor determining the topology of the optimized flow channels and their complexity. The optimized results depend on the initial values of the design variable and the local optimality is only guaranteed within the mesh resolution because, in this paper, the gradient based optimization solver is utilized.

#### 4. Conclusion

We reviewed a way to formulate topology optimization of fluid and fluid-thermal problems by using the density based approach. We also showed the effectiveness of the topology optimization through numerical examples. However, this method has yet to be investigated under more challenging situations including higher Reynolds numbers and a larger ratio of thermal conductivities between the solid and fluid parts.

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#### Tadayoshi Matsumori

Research Fields:

- Optimum Design
- Topology Optimization
- Shape Optimization

Academic Degree: Dr.Eng.

Academic Society:

- The Japan Society of Mechanical Engineers

Award:

- Certificate of Merit for Best Technological Presentation, JSME Computational Mechanics Division Award, 2009



#### Tsuguo Kondoh

Research Fields:

- Computational Fluid Dynamics
- Topology Optimization

Academic Degree: Dr.Eng.

Academic Societies:

- The Japan Society of Mechanical Engineers
- The Japan Society for Aeronautical and Space Sciences
- The Japan Society for Industrial and Applied Mathematics



#### Atsushi Kawamoto

Research Fields:

- Topology Optimization
- Nonlinear Dynamics
- Math Programming

Academic Degree: Ph.D.

Academic Society:

- The Japan Society of Mechanical Engineers



#### Tsuyoshi Nomura

Research Field:

- Topology Optimization

Academic Degree: Ph.D.

Academic Society:

- The Japan Society of Mechanical Engineers

